



EIOPA-BoS-15/035

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# **Technical document regarding the risk free interest rate term structure**

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## Letter of the Executive Director

Solvency II aims at implementing an economic and risk-based supervisory framework in the field of insurance and reinsurance. The framework is built upon three pillars, all equally relevant, that provide for quantitative requirements (Pillar1), qualitative requirements (Pillar 2) and enhanced transparency and disclosure (Pillar 3).

The starting point in Solvency II is the economic valuation of the whole balance sheet, where all assets and liabilities are valued according to market consistent principles.

The risk-free interest rate term structure (hereafter in this letter, risk-free interest rate) underpins the calculation of liabilities by insurance and reinsurance undertakings. EIOPA is required to publish the risk-free interest rate.

This technical document sets out the basis on which it will do so. It is the result of collaboration between EIOPA's members and its staff.

As a default approach, the risk-free interest rate is primarily derived from the rates at which two parties are prepared to swap fixed and floating interest rate obligations. In the absence of financial swap markets, or where information of such transactions is not sufficiently reliable, the risk-free interest rate is based on the government bond rates of the country. The risk-free interest rates are:

- Calculated for different time periods, reflecting that the liabilities of insurance and reinsurance undertakings stretch years and decades into the future.
- Calculated in respect of the most important currencies for the EU insurance market.
- Adjusted to reflect that a portion of the interest rate in a swap transaction (or a government bond) will reflect the risk of default of the counterparty and hence without adjustment would not be risk-free.
- Based on data available from financial markets. For those periods in the more distant future for which data are not available, the rate is extrapolated from the point at which data are available to a macroeconomic long-term equilibrium rate.

An adjustment (the volatility adjustment) is made to the liquid part of the risk-free interest rate in order to reduce the impact of short term market volatility on the balance sheet of undertakings. EIOPA is required to provide, both on a currency and country basis, the size of this adjustment for volatility.

A different adjustment (the matching adjustment) is made in respect of predictable portfolios of liabilities. An undertaking can assign to eligible portfolios assets with fixed cash flows that it intends to hold to maturity. EIOPA is required to provide an estimate of what portion of the spread of such assets above the risk-free interest rate reflects risks not faced by those who hold assets to maturity.

Many of the parameters of the risk-free rates are already determined in legislation. Some choices remain however, and in many cases more than one option is possible. The rationale for the key choices made by EIOPA is in the section 1. Basis for decision of this technical documentation. The choices made by EIOPA, always within the limits set by EU legislation, are designed to secure the following objectives.

### **Replicability**

EIOPA intends the risk-free rate interest rate to be capable of replication by undertakings and other interested parties, through this technical documentation. This will benefit undertakings for their own risk management and other purposes. One consequence of replicability is that the use of so-called "expert judgement" i.e. the exercise of discretion in the regular construction of the risk-free interest rate, has been kept to a minimum.

### **Market consistency**

Whenever possible, data from deep, liquid and transparent financial markets are used to construct the risk-free interest rate. Adopting such a market consistent approach helps foster transparency in insurance markets with a positive impact on understanding and trust, as well as helping create a level playing field by enabling for comparison between undertakings.

### **Solvency II reporting**

The intended frequency of publication of the risk-free interest rate is monthly. Such a frequency will enable undertakings to have a common basis for calculating the value of the financial information they are required to report to their supervisor on a quarterly and annual basis.

### **Stability for insurance undertakings**

EIOPA does not want to exacerbate volatility in the value of liabilities through unwarranted changes to the risk-free interest rate. Changes would naturally have to be justifiable on an EU-wide basis. The experience of those EIOPA members who have already produced risk-free interest rates is however that from time to time the case for change is made. Regardless of any earlier changes, there will also be a more formal stocktake, for example at the point at which the calibration of capital requirements under Solvency II is reviewed.

The risk-free rate interest rate is intended to be published from February 2015, to give undertakings time to prepare. EIOPA does not seek a timescale between publication of the risk-free interest rate and the requirement on undertakings to report that could trigger rapid sale or purchase of assets.

### **Policyholders**

These objectives will benefit policyholders. Replicability, market consistency, Solvency II reporting, and stability for undertakings will make easier the valuation of undertakings and the work of supervisors.

The key components of the risk-free rate are summarised in the table below. They are explained in much greater detail, alongside other components, in the technical documentation.

<b>Component</b>	<b>Approach adopted by EIOPA</b>
Assessment of deep, liquid, transparent financial market information	<ul style="list-style-type: none"> <li>Assessments by each EIOPA member or (for non-EEA currencies) analysis of market interest rates</li> </ul>
Last liquid point (LLP)	<ul style="list-style-type: none"> <li>Euro: residual volume criterion</li> <li>Other EEA currencies: assessment by each EEA member state</li> <li>Non-EEA currencies: EIOPA assessment</li> </ul>
Extrapolation	<ul style="list-style-type: none"> <li>Smith-Wilson method as applied in Long-term Guarantees assessment</li> </ul>
Convergence maturity	<ul style="list-style-type: none"> <li>Euro: 60 years</li> <li>Non-Euro: in general, max(last liquid point +40Y ;60Y)</li> </ul>
Volatility adjustment: calculation of risk correction	<ul style="list-style-type: none"> <li>Calculation in the same manner as the fundamental spread</li> <li>For sovereign debt, based on the long-term average spreads over the basic risk-free interest rates term structure</li> <li>For assets other than sovereign debt, based on the maximum of: <ul style="list-style-type: none"> <li>the long-term average spreads</li> <li>a probability of default and cost of downgrade based on the projection of an average 1-year transition matrix</li> </ul> </li> </ul>
Matching adjustment: calculation of fundamental spread	<ul style="list-style-type: none"> <li>Separate calculation of a probability of default and cost of downgrade based on the projection of an average 1-year transition matrix</li> </ul>

## Legal Notice

1. The Union legislator entrusted EIOPA to lay down technical information on risk-free interest rates with the purpose to allow for the consistent calculation of technical provisions by insurance and reinsurance undertakings under Directive 2009/138/EC.
2. To further reinforce the importance of such EIOPA technical information towards achieving consistency in the calculation of technical provisions, the Union legislator provided for binding effects of this technical information on insurance and reinsurance undertakings, subject to the inclusion of this information into an implementing act of the Commission (Article 77e(2) of the Solvency II Directive).
3. In this respect, EIOPA produced this technical documentation as part of the overall EIOPA framework that is emerging following the performance of EIOPA's responsibilities under Article 77e of Directive 2009/138/EC. The technical information provides in a transparent manner how the relevant risk-free interest rate term structure is derived. To this end, EIOPA has taken account of relevant observations in the financial market.
4. In accordance with recital (23) of the Delegated Regulation, EIOPA technical documentation is part of the technical information set out in Article 77e(1) of the Solvency II Directive in order to achieve a consistent calculation of the technical provisions.
5. The relevant legal references are the following ones:

Recital (29) of Directive 2014/51/EU (Omnibus 2 directive)

*(29) In order to allow for the consistent calculation of technical provisions by insurance and reinsurance undertakings under Directive 2009/138/EC, it is necessary for a central body to derive, publish, and update certain technical information relating to the relevant risk-free interest rate term structure on a regular basis, taking account of observations in the financial market. The manner in which the relevant risk-free interest rate term structure is derived should be transparent. Given the technical and insurance-related nature of those tasks, they should be carried out by EIOPA.*

Article 77e Technical information produced by the European Insurance and Occupational Pensions Authority (Directive 2009/138/EC).

*1. EIOPA shall lay down and publish for each relevant currency the following technical information at least on a quarterly basis:*



*(a) a relevant risk-free interest rate term structure to calculate the best estimate referred to in Article 77(2), without any matching adjustment or volatility adjustment;*

*(b) for each relevant duration, credit quality and asset class a fundamental spread for the calculation of the matching adjustment referred to in Article 77c(1)(b);*

*(c) for each relevant national insurance market a volatility adjustment to the relevant risk-free interest rate term structure referred to in Article 77d(1).*

*2. In order to ensure uniform conditions for the calculation of technical provisions and basic own funds, the Commission may adopt implementing acts which set out, for each relevant currency, the technical information referred to in paragraph 1.*

*Those implementing acts shall make use of that information. Those implementing acts shall be adopted in accordance with the advisory procedure referred to in Article 301(2).*

*On duly justified imperative grounds of urgency relating to the availability of the relevant risk-free interest rate term structure, the Commission shall adopt immediately applicable implementing acts in accordance with the procedure referred to in Article 301(3).*

*3. Where the technical information referred to in paragraph 1 is adopted by the Commission in accordance with paragraph 2, insurance and reinsurance undertakings shall use that technical information in calculating the best estimate in accordance with Article 77, the matching adjustment in accordance with Article 77c, and the volatility adjustment in accordance with Article 77d.*

*With respect to currencies and national markets where the adjustment referred to in paragraph 1(c) is not set out in the implementing acts referred to in paragraph 2, no volatility adjustment shall be applied to the relevant risk-free interest rate term structure to calculate the best estimate.*

Finally the Delegated Regulation contain the following recital

*(23) In order to ensure transparency in the determination of the relevant risk-free interest rate, in accordance with recital 29 of Directive 2014/51/EU, the methodology, assumptions and identification of the data used by the European Insurance and Occupational Pensions Authority (EIOPA) to calculate the adjustment to swap rates for credit risk, the volatility adjustment and the fundamental spread for the matching adjustment, should be published by EIOPA as part of the technical information to be published by virtue of Article 77e(1) of Directive 2009/138/EC.*

6. The technical information has been open to public consultation during the first three weeks of November 2014. Comments received have been processed and resolutions to the comments has been published, except for those comments submitted to EIOPA on confidential basis.
7. In line with Article 77(e) of Directive 2009/138/EC, EIOPA lays down and publishes certain technical information in this technical document. This technical information does not constitute legal advice. EIOPA disclaims all liability in this respect, other than that the information published fulfils the conditions set forth in Article 77(e) of Directive 2009/138/EC.
8. The references to financial data, financial and statistical methodologies, and trademarks mentioned in this technical document are protected by their respective property rights (be they proprietary to EIOPA or third parties). The mention of such information in this technical documentation neither means any change of such rights, nor constitutes any type of explicit or implicit authorization of EIOPA for any use, nor provides any type of opinion of EIOPA in respect of them for purposes other than those proposed in this technical documentation.
9. Whenever reference is made to a (third party) market data provider, the use of the relevant data shall be subject to the terms and conditions of such market data provider, including the relevant disclaimers (as can be consulted on the relevant market data provider's website).
10. References in this Technical Documentation to the Delegated Regulation of Solvency II, refer to COMMISSION DELEGATED REGULATION (EU) 2015/35 of 10 October 2014 supplementing Directive 2009/138/EC of the European Parliament and of the Council on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II), published in Official Journal of the European Union on January 17, 2015

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## **1. Basis for decision**

11. In accordance with recital 23 of the Delegated Regulation, the elaboration of this Technical Documentation has required a number of decisions on the methods, assumptions and inputs to use in the calculation of the relevant risk-free interest rates term structures.
12. For the sake of consistency and efficiency, the first step has been the definition of the framework to steer the decision process. This framework may be summarized in the following principles:
  - a) Respect to the essential elements underpinning the political agreement of the Omnibus II Directive,
  - b) Transparency of all the elements of the process of calculation,
  - c) Replicability of the calculations, which has as a direct consequence the restriction of expert judgement to its minimum possible extent, if any,
  - d) Market consistency, prudent assessment of the technical provisions and optimal use of market information.
13. The following sub-items describe the main decisions adopted, following the order of the topics contained in this Technical Documentation.

### **1.A. General issues**

#### **Financial market data used as inputs**

14. This Technical Documentation identifies the financial market data used as inputs of the calculations.
15. EIOPA keeps unambiguous neutrality regarding the market data providers competing in the market. The reason for selecting market data providers relies only in the high priority given to:
  - a) Firstly, the legal imperative of publishing the concrete figures of the technical information set out in Article 77e of Directive 2009/138/EC,
  - b) Secondly, the full traceability of EIOPA calculations, as part of EIOPA commitment with the principle of transparency,
  - c) Thirdly, the 'replicability' of the process of calculation by those stakeholders wishing to reproduce it,
  - d) And finally in order to put into place an appropriate process of validation, having in mind the impact from different perspectives (e.g. cost of replicability, operational risks,...)

16. According to the four eyes principle two market data sources are used, one for inputs ('direct input provider'), and the other one for validation. EIOPA has decided to use the same 'direct input provider' for swaps and government bonds curves.
17. Regarding yield for corporate bonds and default statistics, EIOPA has opted for reducing the operational risk and dependence, selecting different providers.
18. The selection of these providers should not be understood as EIOPA's preference for them. EIOPA does not consider this selection to be a conclusive element for undertakings when deciding which provider better fits to their needs.

### **Use of market data with maturities less than one year**

19. QIS5 mentioned that the construction of risk-free interest rates term structures adding market data with maturities less than 1 year was not applied at that moment for practical reasons, but this matter would need to be examined further for full Solvency II implementation.
20. EIOPA has decided to start the publication of the relevant risk-free interest rates term structure from 1 year maturity onwards. Instruments below 1 year are not always swaps and the adjustment of their credit risk, among other features, may add an unnecessary complexity. Furthermore, below 1-year rates have a negligible impact on the rates extrapolated with the Smith-Wilson method, and hence a negligible impact on the amount of long-term technical provisions.
21. Nevertheless, EIOPA may investigate market data with maturities less than one year in the Smith-Wilson method for potential use in the future.

### **Methods for the deep, liquid and transparent assessment of the interest rates (DLT assessment)**

22. Based on academic literature and the methods applied by practitioners EIOPA has analysed a set of metrics and criteria commonly used for similar assessments. EIOPA has assessed their applicability for the purposes of setting a conceptual framework for DLT assessment.
23. Having in mind that the National Competent Authorities have the better knowledge of the financial markets of each currency, the DLT assessment of EEA currencies has been made by each National Competent Authority. All National Authorities applied the same methodology and also reported their findings in a common template. Four main findings may be extracted from the set of lessons learnt:

- a) The application of the common conceptual framework should not rely on hard thresholds and should not disregard qualitative information. In particular a number of criteria are inter-linked, and the markets for the same financial instruments for different currencies may present different features.
- b) The DLT assessment is a demanding exercise and, therefore, the frequency of updating the assessment should be carefully considered.
- c) Furthermore, with the exception of crisis situations, frequent violent changes in the outputs of the DLT assessment do not seem plausible. Rather, a plausible future trend will be the development of financial markets and the extension of the market interest rates meeting DLT requirements (i.e. the use of market consistent information).

## **1.B. Basic risk-free interest rates term structure**

### **Credit risk adjustment (CRA)**

- 24. The Delegated Regulation only cover the calculation of the CRA for those currencies with DLT swap markets and overnight swaps markets.
- 25. For currencies where either swaps or overnight swaps markets do not meet DLT requirements or currencies whose risk-free interest rates term structure is based on government bonds rates, EIOPA has privileged the application of the objective criteria described below in section 5, approved by the Board of EIOPA, avoiding any margin for expert judgement at the operational level.
- 26. Furthermore EIOPA is aware of the initiatives in the Union for the development of more transparent financial markets for risk-free financial instruments.

### **Extrapolation method**

- 27. The interpolation, where necessary, and extrapolation of interest rates have been developed applying the Smith-Wilson method.
- 28. This method is of course not the only one possible for extrapolation of the interest rates. In the same manner other methods have their pros and cons, the Smith-Wilson method also has its own features.
- 29. The Smith-Wilson method has been applied during the last years of the development of the Solvency II framework, and in particular in the Fifth Quantitative Impact Study (QIS5) and in the Long-term

Guarantees Assessment (LTGA) that has underpinned the political agreement of the Omnibus II Directive.

30. EIOPA will however carefully monitor market developments, and their influence on the implementation of the Smith-Wilson method.

### **Last Liquid Point (LLP)**

31. Directive 2009/138/EC and the Delegated Regulation set out for the determination of the LLP, the application of DLT requirements and for the Euro a specific recital regarding the residual volume of bonds meeting DLT requirements (aka the '*residual volume criteria*')<sup>1</sup>. For the Euro, the method is precise except for the very specific market data to be used as input.
32. For currencies other than the Euro according to recital 30 of Directive 2014/51/UE (aka the Omnibus II Directive), the choice of the LLP should allow undertakings to match with bonds the cash flows which are discounted with non-extrapolated interest rates in the calculation of the best estimate. In this respect it is relevant to note the current limitation of the information available (e.g. cash flows from insurance and reinsurance obligations).
33. Having in mind all this, for currencies other than the Euro, EIOPA has considered relevant to base the LLP on the results of the DLT assessment, rather than developing the matching criteria at this stage.

### **Convergence point**

34. The Directive explicitly reflects for the Euro a convergence period of 40 years and LLP of 20 years, which is equivalent to assuming that the forward rate will converge to its ultimate level (UFR) from 20+40=60 years maturity onwards.
35. EIOPA adopted an approach for other currencies where the convergence point is the maximum of (LLP+40) and 60 years. This method is considered as the most stable, the less influenced by expert judgement and also the one with lowest impact on the level playing field.

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<sup>1</sup>(21) Under market conditions similar to those at the date of adoption of [Directive 2014/./EU (Omnibus II)], when determining the last maturity for which markets for bonds are not deep, liquid and transparent anymore in accordance with Article 77a Directive 2009/138/EC, the market for bonds denominated in euro should not be regarded as deep and liquid where the cumulative volume of bonds with maturities larger than or equal to the last maturity is less than 6 per cent of the volume of all bonds in that market.

36. In accordance with recital 30 of Directive 2014/51/EU, the selected option keeps the allowance of different outcome for specific cases.
37. This is however conditional on their adequate justification and approval by the EIOPA Board of Supervisors.

### **Ultimate Forward Rate (UFR)**

38. EIOPA published in August 2010 a study justifying the level of the UFR, and concluding in favour of a simplified proposal materialised in a bucketing of the UFR in three levels for all currencies.
39. First, for QIS5 these three UFR levels were adopted. The general UFR was 4.2%, with a lower value of 3.2% for CHF and JPY and a higher value of 5.2% for a few economies with high interest rates. In EIOPA Stress Test 2014, two levels were retained (4.2% and 3.2%)<sup>2</sup>.
40. In the approach applied for the LTGA exercise, only a UFR of 4.2% was tested in accordance with the terms of reference sent to EIOPA by the Trilogue parties. Therefore there was no assessment of the level of the UFR at that moment.
41. In light of the considerations above, EIOPA considers technically sound sticking to QIS5 approach for the UFR (i.e. UFR = 4.2% with exceptions for non-EEA currencies with either long lasting low interest rates or materially higher interest rates).
42. EIOPA considers the analysis performed in QIS5 is still applicable firstly because the UFR is a level of convergence in the very long-term horizon.
43. Secondly because there are still material differences in the behaviour of the interest rates of the main currencies, differences that have remained during last decades and have structural reasons. Therefore revising all UFR downwards or grouping all of them in a single bucket would deviate from the experience published in 2010 and has no support in light of markets behaviour during the last four years.

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<sup>2</sup> EIOPA Stress Test 2014 only stresses EEA currencies + USD + CHF + JPY.

## 1.C. Volatility adjustment (VA) and Matching adjustment (MA)

### Financial market inputs for VA and MA

44. The Delegated Regulation states that the '*manner*' in which the risk correction spread (VA) and the fundamental spread (MA) are calculated should be the same. EIOPA understands that the intention of the sentence '*in the same manner*' in Article 51 is to cover all the elements of the calculation, including the data underlying it. This means that the same decision process should be applied for both the VA and MA.
45. Therefore the decisions of EIOPA in the operationalisation of these calculations have respected this explicit legal intention of avoiding a twofold approach. In particular EIOPA has not used different market default and transition inputs for the VA and MA.
46. EIOPA has gathered inputs on all bonds, securitizations and loans (no differentiation by cash-flow features), using the following granularity: currency, credit quality, duration and economic sector of the issuer. This is based on Article 54 of the Delegated Regulation, the political agreement of the LTGA exercise and the Omnibus II Directive.

### Financial market inputs. Use of market yield indices/curves

47. EIOPA has elaborated a conceptual framework in order to apply to the maximum extent the use of market indices in the calculation of the VA.
48. For this purpose EIOPA applies both the concept of '*yield market index*' (covering both yield curves and indices on yields) and the concept of reference portfolio of '*yield market indices*' (mapping the representative portfolios of the assets which undertakings actually are invested in with a set of yields).
49. In the case of the Euro currency VA, EIOPA has opted for a simplification in the use of indices for central government bonds: the replacement of the calculation based on all the government curves of the members of the Euro zone, by a single curve: ECB curve, annual spot rates, with reference to all members of the Eurozone.
50. For non-Euro currency and for the purpose of the country-specific increase of the VA, the use of yield curves for each issuer of government bonds is necessary given the materially different degrees of home-bias.
51. Finally, in the case of other bonds (e.g. corporate bonds and collateralized bonds, etc.), a major challenge has been the availability of the information with the necessary granularity when considering the full space of information (maturities, ratings, economic sectors), given also the geographical area.



## **Inputs for the calculation of the floor (long-term average spread) for the risk correction (VA) and fundamental spread (MA)**

52. Article 54(3) of the Delegated Regulation sets out:
- The long-term average referred to in Article 77c(2)(b) and (c) of Directive 2009/138/EC shall be based on data relating to the last 30 years. Where a part of that data is not available, it shall be replaced by constructed data. The constructed data shall be based on the available and reliable data relating to the last 30 years. Data that is not reliable shall be replaced by constructed data using that methodology. The constructed data shall be based on prudent assumptions.*
53. The problem is the current lack of 30 years of historical data for swaps and sovereign bonds, for almost all currencies. Furthermore, overnight swap markets (whose short term rates are necessary for the calculation of the credit risk adjustment), were active only since the end of the last century.
54. EIOPA has decided to construct the missing spread data for each currency and maturity using the average of the spread data that is available from 1/1/1985 or, failing that, whenever reliable spread data is first available. In practice, the lack of overnight swap rates has led to consider market data only from January 1999.
55. The same considerations apply to the floor for bonds other than central government and central banks bonds as well, with two further features that increase the practical difficulties:
- a) For most EEA currencies there are no reliable interest rate term structures for corporate bonds.
  - b) For the Euro, the curves currently provided by financial market data providers have a limited history.
56. For the selection of market providers, EIOPA has considered a decision process for central governments and central banks bonds and for other bonds (e.g. corporates), based on a list of criteria for example:
- a) The historical data reconstruction with an impact assessment exercise
  - b) The market information and methodology behind the construction of the market indices (e.g. sovereigns and corporates)
  - c) The granularity (e.g. buckets regarding the maturities, ratings, economic sectors, for bonds other than central governments and central banks).

**Central governments and central banks bonds. Calculation of the long-term average of spreads set out in Art. 77c(2) of Directive 2009/138/EC**

57. Pending on the period of observation, EIOPA has considered whether market data should or should not be adjusted before calculating the average set out in Article 77c(2) of Directive 2009/138/EC.
58. Both in LTGA and EIOPA Stress Test 2014 a simple average was applied.
59. The allowance of adjustments to the simple average means to disregard market observations and embeds the use of material expert judgement. This option lacks legal basement and has been rejected due to the subjective assumptions required.
60. Furthermore, EIOPA believes that assuming a flat curve as reconstructed history (e.g. for the Euro before 1/1/1999) is the most neutral choice as well as being in line with the Directive and political agreement in Omnibus II. The level should be equal to the simple and unadjusted average of the available market spreads.

**Methodology of calculation of the spread before risk correction, for currencies where interest term structures for the determination of  $S_{corp}$  are not available**

61. For most of EEA currencies either there are no available interest rate term structures for the assets relevant to determine  $S_{corp}$ <sup>3</sup> or the number of potential underlying assets to build such curves is rather low. Market data providers only produce corporate yield curves for a few non-EAA currencies (just the most developed financial markets).
62. It should be noted that the option of EIOPA building the curves by itself does not seem feasible – e.g. due to the aforementioned lack of data.
63. In absence of empirical data, EIOPA has decided to apply the following formulas which is based on the LTGA approach:

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<sup>3</sup> According to Article 50 of the Delegated Regulation, *Scorp* denotes the average currency spread on bonds other than governments bonds, loans and securitizations included in the reference portfolio of assets for that currency or country.

$$S_{corp}^X = S_{corp}^{\epsilon} + \kappa \cdot (Y_{rfr}^X - Y_{rfr}^{\epsilon})$$

$$Y_{corp}^X = Y_{corp}^{\epsilon} + (1 + \kappa) \cdot (Y_{rfr}^X - Y_{rfr}^{\epsilon})$$

where  $\epsilon$  denotes the Euro,  $X$  refers to a currency without interest rates term structures for the assets relevant for the spread  $S_{corp}$ ;  $Y_{corp}$  denotes the yield of the respective corporate bonds of the same credit quality;  $Y_{rfr}$  denotes the basic risk-free interest rate; and  $\kappa=0.5$ . The inputs of this formula are maturity dependent according to the information available.

64. On the one hand, the spreads might be better reflected by spreads derived from the basic risk-free rates than using no data. In addition, this method is simple and immediately applicable to all published currencies (where necessary) in a homogeneous manner.
65. Further than its simplicity and traceability, this formula guarantees that for each currency their 'notional' yield curves for corporates will behave -compared to the basic risk-free interest rates term structure- similarly to the main currency where corporate interest rates term structures for the Euro are available for a number of years.
66. Setting  $\kappa = 0.5$  seems the best proxy for a formula to be applied to all relevant currencies. This proxy provides a central estimate and ensures that differences with the more accurate and complex calculation is reduced to the maximum extent possible using a simple and implementable approach.

**Bonds other than central government bonds. (Article 50(d)).  
Methodology for the calculation of the risk correction and fundamental spread.**

67. Two aspects are considered. Firstly the selection of the inputs based on yield market indices/curves. Secondly the calculation of the probability of default (PD) and cost of downgrade (CoD). The input referred to the last observed spreads should be consistent with the financial market data used in the calculation of the long-term average of spreads.

**Granularity according to the full dimensions (maturities, ratings, economic sectors) and class of assets.**

68. A minimum granularity regarding the economic sectors has been adopted in the right dimensions (maturities, ratings, economic sectors) in order to adequately capture the asymmetric behaviour of spreads (e.g. financials and non-financials).

## 2. Process and controls. Governance and control of the process of calculation and publication

69. EIOPA has established internal governance arrangements in order to define the essential elements of the **operational framework** such as:
- i) The period of time after which the technical information shall be published,
  - ii) Definition of the functions involved,
  - iii) The resources necessary for running the process and the registers and logs for recording,
  - iv) Internal controls to safeguard the process used built on 'four eyes' principle,
  - vi) The frequency of activities, in particular audits, reviews and internal controls,
  - viii) Definition in a limitative manner of the areas where expert judgement in the process is allowed (e.g. some areas of the DLT assessment). In that case, the documentation of the expert judgement includes its content, link to the authorized scope, validation, internal control and log of escalation, in order to ascertain that, in accordance with the EIOPA regulation, such expert judgement is independently exercised, it acts in the interest of the Union, enhances the protection of policyholders and fosters a level playing field of the EU insurance market,
  - ix) Definition of the specific process to follow where after the publication of the technical documentation, new information might advise the review of the technical information already published. EIOPA rules on public consultation will apply to the review of the technical documentation,
  - x) Contingency plans for continuing the publication of the technical information in case of unexpected events,
  - xi) Rules in order to record, store and report exceptional events in the development of any of the steps of the process(process events, IT events, financial market data events, etc.),
  - xii) Establishment of an oversight function and of a control function ensuring that the technical information is provided and published or made available in accordance with the methodology, assumptions and inputs approved by EIOPA.
70. The EIOPA framework regarding code of conduct and conflict of interests applies to all the persons involved in the process in any function. All these persons have to declare and sign the relevant documentation at least every year, and as soon as any factual or

potential, current or foreseeable, conflict of interest appears or may appear.

71. EIOPA has not approved and does not envisage approving, the outsourcing of any function or activity of the process for the calculation and publication of the technical information, other than the collection of data of financial markets from generally used financial providers, and the outsourcing applied to some parts of the IT systems of EIOPA.

### **3. Data sources for the inputs from financial markets**

#### **3.A. Financial market data providers**

72. According to EIOPA's approach to its own operational risks, the calculation of the technical information set out in Article 77e of Directive 2009/138/EC should not over-rely on a single market source.
73. A first way to ensure this would be to derive each input using data obtained from a range of providers. A second alternative would be to calculate a given input based on data from a single market provider, but to use different providers for different inputs or functions, under the condition that all sources are sufficiently consistent.
74. As a general rule EIOPA has opted for the second of these options, on the basis that an application of the first option to all inputs would introduce additional complexity and increase the operational risks, without providing material benefits compared to the second alternative.
75. EIOPA has no evidence of the superiority of a concrete market data provider. The choices of market data providers included in the technical documentation are disclosed only for the purposes of transparency (recital 23 of the Delegated Regulation).
76. In accordance with recital (23) of the Delegated Regulation, EIOPA technical documentation is part of the technical information set out in Article 77e(2) of the Solvency II Directive in order to achieve a consistent calculation of the technical provisions.
77. The following providers have been used (see subsections below for detail):
  - a. Swaps and overnight interest rates: Inputs will be based on Bloomberg.
  - b. Sovereign bonds: Inputs will be based on Bloomberg.
  - c. Bonds other than government bonds or other assets: Inputs will be based on Markit – iBoxx indices.

d. Default statistics: Inputs will be based on Standard & Poors statistics (see the annex to this section).

78. The market data inputs will be analysed under the relevant review process according to Section 1, vi)

### **3.B. Selection of the currencies**

79. EIOPA applies the following criteria to select the currencies (and countries for the country specific increase of the volatility adjustment) whose technical information is published:

As a general principle all currencies and countries (for the country specific increase of the volatility adjustment) of the EEA are included,

For non-EEA currencies, EIOPA intends to publish the technical information of those currencies where EIOPA has evidence on their materiality for the EU insurance sector, and furthermore reliable financial market data are publicly available and adequate to perform the necessary calculations.

80. The following subsection contains in Table 1 the list of currencies and countries whose technical information is currently published by EIOPA.

### **3.C. Selection of market rates**

81. The construction of the basic risk-free rates term structures is based on the financial instruments set out in Article 44 of the Delegated Regulation (swaps and/or government bonds). EIOPA is aware of the initiatives in the Union to develop in the future DLT instruments with observable traded prices.

82. EIOPA applies the financial references in the table below from the market data provider selected. Prices are mid prices.

83. The last column of the table specifies whether the financial instruments applied are either swaps or government bonds. For a clear identification of swaps, the floating is also included.

84. In the process of calculation of the basic risk-free interest rates term structures, the tickers for the government bonds are used only for those currencies with 'GVT' in the last column. The inputs to the process of calculation of the volatility and matching adjustments regarding sovereign bonds are also based on the information referring to the table below.

**Table 1. Identification of the financial instruments used for the derivation of the basic risk-free interest rate term structures**

Country	ISO3 166	ISO4 217	Swaps_Ticker	Swaps freq	Swap_Float_ Ticker	Govts_Ticker_Id	Govts/ Swaps
Euro	EUR	EUR	EUSA CMPN Curncy	1	EUR006M Index	ECB curve all governments-spot	SWP
Austria	AT	EUR	EUSA CMPN Curncy	1	EUR006M Index	I063 CMPN Index	SWP
Belgium	BE	EUR	EUSA CMPN Curncy	1	EUR006M Index	I006 CMPN Index	SWP
Bulgaria (*)	BGN	BGN	EUSA CMPN Curncy	1	EUR006M Index	F999 CMPN Index	SWP
Croatia	HRK	HRK				I369 CMPN Index	GVT
Cyprus	CY	EUR	EUSA CMPN Curncy	1	EUR006M Index	I261 CMPN Index	SWP
Czech Rep.	CZK	CZK	CKSW CMPN Curncy	1	PRIB06M Index	I112 CMPN Index	SWP
Denmark (*)	DKK	DKK	EUSA CMPN Curncy	1	EUR006M Index	I011 CMPN Index	SWP
Estonia	EE	EUR	EUSA CMPN Curncy	1	EUR006M Index		SWP
Finland	FI	EUR	EUSA CMPN Curncy	1	EUR006M Index	I081 CMPN Index	SWP
France	FR	EUR	EUSA CMPN Curncy	1	EUR006M Index	I014 CMPN Index	SWP
Germany	DE	EUR	EUSA CMPN Curncy	1	EUR006M Index	I016 CMPN Index	SWP
Greece	GR	EUR	EUSA CMPN Curncy	1	EUR006M Index	I156 CMPN Index	SWP
Hungary (*)	HUF	HUF	HFSW CMPN Curncy	1	BUBOR06M Index	I165 CMPN Index	GVT
Iceland (*)	ISK	ISK				I328 CMPN Index	GVT
Ireland	IE	EUR	EUSA CMPN Curncy	1	EUR006M Index	I062 CMPN Index	SWP
Italy	IT	EUR	EUSA CMPN Curncy	1	EUR006M Index	I040 CMPN Index	SWP
Latvia	LVL	EUR	EUSA CMPN Curncy	1	EUR006M Index	I315 CMPN Index	SWP
Liechtenstein	LIC	CHF	SFSW CMPN Curncy	1	SF0006M Index		SWP
Lithuania	LTL	EUR	EUSA CMPN Curncy	1	EUR006M Index	I341 CMPN Index	SWP
Luxembourg	LU	EUR	EUSA CMPN Curncy	1	EUR006M Index		SWP
Malta	MT	EUR	EUSA CMPN Curncy	1	EUR006M Index		SWP
Netherlands	NL	EUR	EUSA CMPN Curncy	1	EUR006M Index	I020 CMPN Index	SWP
Norway (*)	NOK	NOK	NKSW CMPN Curncy	1	NIBOR6M Index	I078 CMPN Index	SWP
Poland	PLN	PLN				I177 CMPN Index	GVT
Portugal	PT	EUR	EUSA CMPN Curncy	1	EUR006M Index	I084 CMPN Index	SWP
Romania (*)	RON	RON	RNSW CMPN Curncy	1	BUBR3M Index	C489 CMPN Index	SWP
Russia	RUB	RUB	RRSWM CMPN Curncy	1	MOSKP3 Index	I326 CMPN Index	SWP
Slovakia	SK	EUR	EUSA CMPN Curncy	1	EUR006M Index	I256 CMPN Index	SWP
Slovenia	SI	EUR	EUSA CMPN Curncy	1	EUR006M Index	I259 CMPN Index	SWP
Spain	ES	EUR	EUSA CMPN Curncy	1	EUR006M Index	I061 CMPN Index	SWP
Sweden	SEK	SEK	SKSW CMPN Curncy	1	STIB3M Index	I021 CMPN Index	SWP
Switzerland	CHF	CHF	SFSW CMPN Curncy	1	SF0006M Index	I082 CMPN Index	SWP
United Kingdom	GBP	GBP	BPSW CMPN Curncy	2	BP0006M Index	I022 CMPN Index	SWP
Australia	AUD	AUD	ADSW CMPN Curncy	2	BBSW6M Index	I001 CMPN Index	SWP
Brazil	BRL	BRL				I393 CMPN Index	GVT
Canada	CAD	CAD	CDSW CMPN Curncy	2	CDOR03 Index	I007 CMPN Index	SWP
Chile	CLP	CLP	CHSWP CMPN	2	CLICP Index	I351 CMPN Index	SWP

			Currency				
China, People's Rep	CNY	CNY	CCSWO CMPN Currency	4	CNRR007 Index	F020 CMPN Index	SWP
Colombia	COP	COP	LSWD CMPN Currency	4	DTF RATE Index	I217 CMPN Index	SWP
Hong Kong	HKD	HKD	HDSW CMPN Currency	4	HIHD03M Index	I095 CMPN Index	SWP
India	INR	INR				F123 CMPN Index	GVT
Japan	JPY	JPY	JYSW CMPN Currency	2	JY0006M Index	I018 CMPN Index	SWP
Malaysia	MYR	MYR	MRSWQO CMPN Currency	4	KLIB3M Index	I196 CMPN Index	SWP
Mexico	MXN	MXN				I251 CMPN Index	GVT
New Zealand	NZD	NZD	NDSW CMPN Currency	2	NFIX3FRA Index	I049 CMPN Index	SWP
Singapore	SGD	SGD	SDSW CMPN Currency	2	SORF6M Index	I107 CMPN Index	SWP
South Africa	ZAR	ZAR	SASW CMPN Currency	4	JIBA3M Index	F262 CMPN Index	SWP
Korea, South	KRW	KRW	KWSWO CMPN Currency	4	KWCDC Index	I173 CMPN Index	SWP
Taiwan	TWD	TWD				F126 CMPN Index	GVT
Thailand	THB	THB	TBSWO CMPN Currency	2	THFX6M Index	F122 CMPN Index	SWP
Turkey	TRY	TRY	TYSWV3 CMPN Currency	1	TRLIB3M Index	F924 CMPN Index	SWP
USA	USD	USD	USSW CMPN Currency	2	US0003M Index	I111 CMPN Index	SWP

Note: Bloomberg's identifiers. Prices PX\_LAST.

85. Specific cases are:

- (a) The Hungarian currency, where the observed market interest rates for 1 year and 2 years maturities are based on swaps, while from 3 years onwards the observed interest rates are based on government bonds. This shift from the use of swap to the use of government bond data is referred to as '*switching*'. For simplicity, the credit risk adjustment of the Hungarian government bond rates is applied to all maturities.
- (b) The Norwegian currency, whose 1 year interest rate is based on swaps with floating NIBOR 03 months, while the rest of interest rates are based on NIBOR 06 months.
- (c) For those non Euro countries with contracts where the benefits guaranteed to the policy holders are valued in Euro while the payments (including the evolutions of the exchange rate) are in the local currency, the term structure is derived on the basis on the interest rates denominated in the local currency.
- (d) The Icelandic curve is observed in ticker YCG328 CMPN Index.
- (e) For the Romanian currency the use of swap curve is still under assessment.
- (f) For Bulgarian and Danish markets the EUR curve has been used because these two EEA currencies meet the legal conditions to be considered as pegged to the Euro.



## Basic risk-free interest rate term structure

### 4. Selection of financial instruments and assessment against the deep, liquid and transparent criteria.

#### 4.A. Legal framework

86. The assessment of depth, liquidity and transparency (DLT) of the relevant observable market interest rates has been developed in accordance with Article 77a of Directive 2014/51/EU, and Recital 21 and Articles 1(32, 33, 34), 43, 44, and 46 of the Delegated Regulation.
87. The inputs for the DLT assessments are market data on interest rate swap rates, government bond rates and corporate bond rates. These are obtained from market data providers whose services are also available to insurance and reinsurance undertakings.
88. The output of the DLT assessment is a list, for each currency, of the maturities that are considered DLT and the identification of the last maturity for which rates can be observed in DLT markets (section 6.B refers to the assessment of the last liquid point(LLP)).

#### 4.B. Conceptual framework for EEA currencies

89. The DLT assessment for EEA currencies is initially carried out by the relevant National Competent Authorities.
90. At a second step, EIOPA has a process in place aimed at ensuring homogeneity across national assessments and preserving a level playing field.
91. The following currencies exist in the European Economic Area (EEA): EUR (BE, DE, EE, FI, FR, GR, IE, IT, LT, LU, LV, MT, NL, AT, PT, SK, SI, ES, CY), CHF (LI), NOK (NO), ISK (IS), BGN (BG), DKK (DK), HRK (HR), PLN (PL), RON (RO), SEK (SE), CZK (CZ), HUF (HU), GBP (UK). The EEA consists of the 28 EU countries, together with Liechtenstein (LI), Iceland (IS) and Norway (NO). During 2013, EIOPA carried out a DLT assessment for EEA currencies. This first assessment was subject to some limitations (in particular, the incompleteness of the legal framework at that stage, and lack of some data). The results shown in this documentation are those of the 2013 assessment including the updates from a review during the last quarter 2014. EIOPA intends to fully update the DLT assessment of EEA currencies in the first half of 2015.

92. The entries of the table below identify the instrument used: S=Interest rate swap, B=government bond, «empty»=no DLT markets for this maturity available. The last non-empty entry defines the LLP. No market data beyond the LLP is used. Hence, no further entries are shown in the table, even if single maturities beyond the LLP might be considered as meeting DLT criteria.
93. As explained in the Annex to this section, the DLT output parameters (the LLP and the reference instrument used for each maturity) do not need to be updated with the same frequency as the curves are published by EIOPA. However, although the DLT assessment does not need to be carried out on a continuous basis, EIOPA is aware of the need for appropriate monitoring of market developments that may have an impact on the DLT assessment.
94. EIOPA only uses the market data inputs for integer maturities from one year onwards.
95. Where there is no market information for a currency as of the date of calculation, the market information of the preceding trading day is used.

**Table 2. EEA currencies: Financial instruments used for the derivation of the basic risk-free interest rate term structures**

	EUR	LIC <sup>4</sup>	NOK	PLN	ISK	HRK	RON	SEK	CZK	HUF	GBP
1Y	S	S	S	B			S	S	S	S	S
2Y	S	S	S	B	B	B	S	S	S	S	S
3Y	S	S	S	B		B	S	S	S	B	S
4Y	S	S	S	B		B	S	S	S	B	S
5Y	S	S	S	B		B	S	S	S	B	S
6Y	S	S	S	B	B		S	S	S	B	S
7Y	S	S	S	B		B	S	S	S	B	S
8Y	S	S	S	B			S	S	S	B	S
9Y	S	S	S	B			S	S	S	B	S
10Y	S	S	S	B			S	S	S	B	S
11Y	S	S			B						S
12Y	S	S							S		S
13Y	S	S									S
14Y	S	S									S
15Y	S	S		B					S	B	S
16Y to 19Y	S	S									S
20Y <sup>5</sup>	S	S									S
21 to 24Y											S
25Y		S									S
26 to 29Y											S
30Y											S
35-40-45-50Y											S

<sup>4</sup> For Liechtenstein the DLT assessment of the Swiss Franc applies

<sup>5</sup> For the Euro, DLT assessment has considered Recital 21 of the Delegated Regulation

**Table 3. DLT assessment for government bonds for EEA currencies whose risk-free interest rate term structures are based on government bonds rates. (1 = DLT , 0 = non-DLT)**

Country	ISO3166	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Croatia	HRK	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Hungary	HUF	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0
Iceland	ISK	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Poland	PLN	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0

For the Hungarian currency, the 1 and 2 years rates are those observed in the Hungarian swap market.

**Table 4. DLT assessment for swaps for EEA currencies whose risk-free interest rates term structure is based on swaps. (1 = DLT , 0 = non-DLT)**

Country	ISO3166	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	
Euro	EUR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	
Bulgaria	BGN	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	
Czech Republic	CZK	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Denmark	DKK	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
Hungary	HUF	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Liechtenstein	LIC	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Norway	NOK	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Romania	RON	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sweden	SEK	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
United Kingdom	GBP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

For the Euro and currencies pegged to the Euro, the last liquid point is 20 years.

For the Bulgarian and Danish currencies, the DLT assessment is based on the Euro curve.

For Liechtenstein, the Swiss Franc curve applies.

#### 4.C. Conceptual framework for non-EEA currencies

96. The DLT assessment for non-EEA currencies is carried out using a specific approach based on the empirical evidence provided by market information on the behaviour of the relevant rates. The empirical evidence is assessed using a twofold approach (see the Annex to this subsection for a more detailed explanation):
- a. volatility analysis;
  - b. analysis of the bid-ask spread. This analysis is carried out for all currencies using both the observed bid-ask spread and also the approximation developed through the Roll measure, as applied in the EBA's report<sup>6</sup> on (extremely) High Quality Liquid Assets (HQLA).
97. The two aforementioned approaches are supported by three toolkits:
- a. Chart analysis, decomposed into analysis of volatility and analysis of bid-ask spread with the Roll measure;
  - b. Quantitative analysis;
  - c. Qualitative analysis.
98. In the case that these approaches do not lead to conclusive results, it is considered that there is no evidence of the DLT nature of the interest rate for the maturity and currency analysed. The interest rate for that maturity and currency is disregarded as input.
99. For the time being, according to the applicable regulations, EIOPA publishes four non-EEA currencies whose risk-free interest rate term structures are based on government bond rates (i.e. there is no evidence that any swap rates meet the DLT requirements).

**Table 5. DLT assessment for non-EEA currencies whose risk-free interest rate term structures are based on government bonds  
(1 = DLT , 0 = non-DLT)**

Country	ISO3166	1	2	3	4	5	6	7	8	9	10	...	15	...	20
Brazil	BRL	1	1	1	1	1	1	1	1	1	1	...	0	...	0
India	INR	1	1	1	1	1	1	1	1	1	1	...	0	...	0
Mexico	MXN	1	1	1	1	1	1	1	1	1	1	...	1	...	1
Taiwan	TWD	1	1	1	1	1	1	1	1	1	1	...	0	...	0

<sup>6</sup><http://www.eba.europa.eu/documents/10180/16145/EBA+BS+2013+413+Report+on+definition+of+HQLA.pdf>

Table 6 shows the DLT assessment for non-EEA currencies whose risk-free interest rate term structures are based on swaps.

**Table 6. DLT assessment for swaps for non-EEA currencies whose risk-free interest rate term structures are based on swaps(1 = DLT , 0 = non-DLT)**

Country	ISO 3166	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	40	45	50		
Russia	RUB	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Switzerland	CHF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0
Australia	AUD	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0
Canada	CAD	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0
Chile	CLP	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
China, P.Rep	CNY	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Colombia	COP	1	1	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hong Kong	HKD	1	1	1	1	1	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Japan	JPY	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
Malaysia	MYR	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
New Zealand	NZD	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
Singapore	SGD	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
South Africa	ZAR	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Korea, South	KRW	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0
Thailand	THB	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Turkey	THB	1	1	1	1	1	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
United States	USD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1

From 20 years onwards, only the rates for the maturities shown in the table above are applied.

#### **4.D. Currencies without DLT financial instruments**

100. For those currencies where EIOPA does not publish the technical information set out in Article 77e of Directive 2009/138/EC the methodology described in this document should be applied.
101. In case of lack of reliable financial market data to apply the methodology, it is expected that undertakings, the relevant EEA supervisor and the supervisor of the corresponding country will have a dialogue in order to select an approach among the most widely applied techniques.
102. For that purpose the use of the basic risk-free interest rates term structures of economies sufficiently similar or inter-linked, may be an option, provided that any adjustment to the term structure used as reference is made under a prudent and objective process, and it is compatible with the methodology described in this document

## **5. Credit risk adjustment and currency adjustment for currencies pegged to the Euro**

### **5.A. Legal framework**

103. The calculation of the credit risk adjustment has been developed in accordance with Recital (20) and article 45 of the Delegated Regulation. The calculation of the currency adjustment for currencies pegged to the Euro has been developed in accordance with Article 48 of the Delegated Regulation.

### **5.B. Conceptual framework**

104. The *credit risk adjustment* (CRA) is applied as a parallel downward shift of the market yields observed for those maturities up to the last liquid point (LLP).

105. The credit risk adjustment (CRA) is applied to the observed par swap rates before deriving zero coupon rates. In the case of risk-free interest rate term structures based on government bond rates, the rates used as '*observed market rates*' are already zero coupon rates. The credit risk adjustment is applied to those government bonds rates.

106. The *currency adjustment for currencies pegged to the Euro*(CPEA) is applied as a second downward shift in addition to the CRA.

107. Either the *credit risk adjustment* or the *currency adjustment for currencies pegged to the Euro* may lead to negative interest rates (i.e. there is no floor for the credit and currency adjusted rates).

### **5.C. Process of calculation of the credit risk adjustment**

108. The calculation of the CRA considers three possible situations, each of which is successively described below.

#### *First situation*

109. In the first situation, the risk-free interest rate term structure is based on swap rates and the relevant overnight indexed swap (OIS) rate meets the DLT requirements.

110. In this case the formula prescribed in Article 45 of the Delegated Regulation for the credit risk adjustment applies, with the following methodological conventions:



- a. The maturity of the OIS rate used to derive the CRA is consistent with the tenor of the floating legs of the swap instruments used to derive the term structure.

For example, the risk-free interest rate term structure for the Swiss currency is based on swaps with floating leg that refers to the six month IBOR. Consistently with this, the OIS rate used in the CRA calculation is the 6 month Swiss OIS rate.

In the case of the Swedish currency, the risk-free interest rate term structure is based on swaps with floating leg that refers to the three month IBOR, and consequently the OIS rate used in the CRA calculation is the 3 month Swedish OIS rate.

- b. For the Euro, the OIS rate to be used is the 3-month rate, as specified in Recital 20 of the Delegated Regulation.
- c. The calculations, and in particular the average, set out in the aforementioned legal provisions are based on daily historical series for the last twelve months.

111. In cases where market data is missing for either the swap instrument or for the relevant OIS rate, the missing data are linearly interpolated. That said, if both terms of the comparison are missing for more than 20% of the business days during the preceding year, it is considered that DLT requirements are not met. Therefore the third method described in this subsection applies.
112. The average is a simple average calculated giving equal weight to all of the observations.

#### Second situation

113. The second situation considered for the calculation of the CRA concerns EEA currencies whose risk-free interest rate term structures are based on government bond rates (table 3 above). For these currencies, the same CRA as for the Euro applies.

#### Third situation

114. In the third situation, for the remainder of currencies where there is not a DLT OIS market<sup>7</sup>, the following method applies:
  - a. A ratio is calculated comparing the sum of the interest rates for the currency for maturities from 1 to 10 years (numerator), to

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<sup>7</sup> Currencies listed in tables 5 and 6, except those listed below in table 7.

the sum of the interest rates for the euro and the same maturities (denominator). Only maturities meeting DLT requirements for both currencies are considered.

- b. The ratio is applied to the CRA for the Euro before the application of the corridor (i.e. after applying the 50% factor and before applying the corridor).
- c. The credit risk adjustment for the currency is the result of applying to the output of step (b), the corridor of [10-35] basis points set out in the Delegated Regulation.

115. Specific cases not covered above are as follows:

- a. Hungarian currency: The CRA for Hungarian government bonds (i.e the CRA for the Euro, according to the second situation above) is applied to all market rates used as input, including the two first maturities based on interest rate swap rates.
- b. Liechtenstein: CRA for the Swiss currency applies.
- c. Norwegian currency: CRA for the Swedish currency applies.

116. For all currencies, the corridor for the CRA set out in Article 45 of the Delegated Regulation applies [10-35 bp], and the CRA is rounded to the nearest integer basis points.

#### **5.D. Data sources for the credit risk adjustment**

117. The following table lists the currencies with DLT overnight swaps markets (i.e. the first situation described above applies).

**Table 7. Currencies whose overnight swap rate is relevant for the calculation of the credit risk adjustment**

Country	ISO 4217	Bloomberg ticker (PX_LAST)	
Euro	EUR	EUR003M Index	EUSWEC CMPN Curncy
Sweden	SEK	STIB3M Index	SKSWTNC CMPN Curncy
Switzerland	CHF	SF0006M Index	SFSWTF CMPN Curncy
United Kingdom	GBP	BP0006M Index	BPSWSF CMPN Curncy
Canada	CAD	CDOR03 Index	CDSOC CMPN Curncy
Japan	JPY	JY0003M Index	JYSOC CMPN Curncy
United States	USD	US0003M Index	USSOC CMPN Curncy

## 5.E. Adjustment for currencies pegged to the Euro

118. According to Article 48 of the Delegated Regulation, the Danish and the Bulgarian currencies can be considered as currencies pegged to the Euro for the purpose of deriving the basic risk-free interest rate term structures
119. The calculated adjustment is based on the following formula<sup>8</sup>

$$CPEA = -factor \cdot \frac{BE}{SCR(0)} \cdot \frac{LAC}{Duration} \cdot \frac{RM}{TP}$$

where the parameter "factor" is equal to the currency risk shock for the relevant currencies (Danish and Bulgarian) towards the euro as defined in the implementing technical standard with regard to the adjusted factors to calculate the capital requirement for currency risk for currencies pegged to the euro according to Article 109a(2)(c) of Directive 2009/38/EC.

120. The rationale of the formula starts in [ $factor \cdot BE$ ] which is meant to reflect the currency risk on the total amount of liabilities i.e. assuming that none of these are hedged towards the Euro. The risk is then corrected for any loss absorbing capacity of TP (LAC) and compared to the SCR which gives the proportion of currency risk to SCR. This is translated into a risk margin change by taking into account the proportion of the risk margin to TP. Finally this is translated into a change in the discount curve by using the duration of TP as a driver.
121. Having in mind the nature of the adjustment and in order to facilitate undertakings their management of interest rates risks, EIOPA will not update the values of the currency adjustment on regular basis, but only when there is available evidence showing up material deviations.
122. The calibration of the currency adjustment is as follows<sup>9</sup>:
- for the Bulgarian currency amounts to 5basis points.
  - for the Danish currency amounts to 1 basis point.

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<sup>8</sup> The adjustment is a consequence of the following expression

$$\frac{-factor \cdot BE \cdot LAC}{SCR(0)} \cdot RM = TP \cdot Duration \cdot CPEA$$

<sup>9</sup> The calculations have been performed based on data of EIOPA Stress Test 2014. The formula applied for the purpose has been the following one:

$$-factor \cdot \left( \frac{BE_{Life} + BE_{NL}}{SCR(0)_{Life} + SCR(0)_{NL}} \right) \left( \frac{BE_{Life} \cdot LAC_{Life} + BE_{NL} \cdot LAC_{NL}}{BE_{Life} \cdot Duration_{Life} + BE_{NL} \cdot Duration_{NL}} \right) \left( \frac{RM_{Life} + RM_{NL}}{TP_{Life} + TP_{NL}} \right)$$

## **6. Extrapolation and interpolation**

123. The parameters used for the extrapolation, except for the financial market data itself, will be assumed to be stable. This includes the ultimate forward rate (UFR), the convergence tolerance, the convergence period, the last liquid point (LLP), and the support of the selected extrapolation method, i.e. the maturities considered to meet the DLT requirements. Changes in the DLT assessment measures will not necessarily translate into changes in the parameters used for extrapolation, the LLP or the support of the selected extrapolation method. Therefore, the DLT assessment will be less frequent than the publication of the risk-free interest rate term structures.
124. EIOPA will publish the interest rates for integer maturities from one year maturity onwards.

### **6.A. Extrapolation method**

125. For each currency the basic risk-free interest rate term structure is constructed from a finite number of DLT market data points, corresponding to the reference instruments. Both the interpolation between these data points, where necessary, and the extrapolation beyond the last liquid point shall be done using the Smith-Wilson methodology. This methodology is described in the Annex to this section.
126. The control input parameters for the interpolation and extrapolation process are the ultimate forward rate, the convergence period and the convergence tolerance. A cash-flow matrix is derived from market interest rate data. The Smith-Wilson interpolation method takes care that the implied present value discount function exactly agrees with the empirical data on the observed maturity node points.
127. If the reference instruments are swap rates, the market interest rates to be used as inputs are the swap par rates after deduction of the credit and currency risk adjustments described in section 5. If the reference instruments are zero coupon government bonds, the market interest rates to be used as inputs are the zero coupon rates after deduction of the credit and currency risk adjustments described in section 5.

## **6.B. Last liquid point for EEA currencies**

128. Recital 21 of the Delegated Regulation defines a method (referred to as the '*residual volume criterion*') to calculate the LLP, which applies for the Euro only. The '*residual volume criterion*' considers all bonds in the market, including corporate bonds. Having computed the outstanding bond volume for each maturity, the sum of the outstanding bond volumes for all maturities  $\geq M$  is computed. The smallest maturity  $M$  for which that sum drops below 6% is considered to no longer meet the DLT criteria. For the Euro, this gives an LLP of 20 years.
129. For all other EEA currencies, the LLP has been chosen according to the results of the DLT assessment. In the table 2 under section 4, the LLP is the first entry for which all longer maturities are empty.
130. For non-EEA currencies, no stable method has been identified for the time being that could be mechanically applied to further update the DLT assessment.
131. The LLP is considered to be generally stable over time, which means that once the first full DLT assessment is done, it is not necessary to pursue a continuous repetition of the full assessment, but, rather an appropriate monitoring of financial markets. It is thus expected that the update of the LLP will be carried out at a lower frequency than the publication of RFR curves (e.g. yearly basis).

## **6.C. Last liquid point for non-EEA currencies**

132. For these currencies, the last liquid point (LLP) is considered to be the longest maturity that meets the DLT requirements.

## **6.D. Ultimate forward rate**

133. The assessment of the UFR is based only on estimates of expected inflation and of the long-term average of short term real rates. For the risk-free interest rate term structure the assumptions are as follows:
- a) 3.2% for Swiss currency and Japanese currency;
  - b) 4.2% for EEA currencies and those non-EEA currencies not explicitly mentioned elsewhere;
  - c) 5.2% for the Brazilian, Indian, Mexican, Turkish and South African currencies.
134. Annex 6.D contains the rationale for these choices. The calibration mentioned in the Annex does not pre-empt any decisions regarding future recalibrations of this parameter.

## **6.E. Convergence point**

135. The convergence point is the maximum of (LLP+40) and 60 years (i.e. the convergence period is the maximum of (60-LLP) and 40 years).
136. The parameter alpha that controls the convergence speed, is set as the lowest value that produces a curve reaching the convergence tolerance of the UFR by the convergence point. The convergence tolerance is set at 1 bp. A lower bound for alpha is set at 0.05. The convergence criterion is assessed by EIOPA with a scanning procedure with six decimals precision for alpha.
137. In accordance to recital 30 of Directive 2014/51/EU, there is open room for specific cases, although conditioned to their adequate justification and approval by EIOPA Board.
138. EIOPA Board of Supervisors, considering the characteristics of the Swedish bond market, has approved a convergence period for the Swedish currency of ten years.

## Section on the volatility and matching adjustment

### 7. Introduction: Conceptual Framework.

139. According to Article 77e of Directive 2009/138/EC:  
*EIOPA shall lay down and publish for each relevant currency the following technical information at least on a quarterly basis:*
- ...
- (b) for each relevant duration, credit quality and asset class a fundamental spread for the calculation of the matching adjustment referred to in Article 77c(1)(b);*
- (c) for each relevant national insurance market a volatility adjustment to the relevant risk-free interest rate term structure referred to in Article 77d(1)*
140. This part of the technical documentation describes how EIOPA derives the technical information mentioned above, in accordance with Articles 77b, 77c and 77d of Directive 2009/138/EC and Articles [49 to 54] of the Delegated Regulation.
141. The choices made by EIOPA in deriving the Risk Correction and the Fundamental Spreads are
- a. The range and granularity of asset classes, credit quality steps and durations for which the Risk Correction and the Fundamental Spreads are published;
  - b. Treatment of currencies for which source data is not available;
  - c. Source data for the probability of default (PD) calculation;
  - d. Method of deriving PD from source data;
  - e. Source data for the cost of downgrade (CoD) calculation;
  - f. Method of deriving CoD from source data;
  - g. Source data for the long-term average of spreads (LTAS) calculation;
  - h. Method of constructing data to give a 30 year spread history.
142. As well as the methodology to derive the volatility adjustment and the fundamental spread for the matching adjustment, these choices are explained in the sections below and a full description of all methodologies is presented.

## 7.A. Conceptual framework of the volatility adjustment

### 7.A.1. Currency volatility adjustment

143. For each relevant currency, a volatility adjustment is an adjustment to the relevant risk-free interest rate term structure based on 65% of the risk-corrected currency spread between the interest rate that could be earned from bonds, loans and securitisations included in a reference portfolio for that currency, and the rates of the relevant basic risk-free interest rate term structure.
144. In order to determine a currency volatility adjustment, EIOPA uses the following inputs:
- a. A currency **representative portfolio<sup>10</sup> of bonds**, securitisations, loans (including mortgage loans), equity and property covering the best estimate of obligations denominated in that currency, based on insurance market data collected by the means of the regulatory reporting;
  - b. A currency **reference portfolio of 'yield market indices'** based on the aforementioned representative portfolio. The expression 'yield market indices' covers in this section both yield curves and indices on yields.
145. Those inputs are used to calculate the following outputs:
- a. the **currency spread  $S$**  between the interest rate derived from the reference portfolio of indices and the rates of the relevant basic risk-free interest rate term structure;
  - b. **the portion of the currency spread  $S$ , denoted  $RC$  for risk correction**, which shall correspond to "the portion of the spread that is attributable to a realistic assessment of expected losses, unexpected credit risk or any other risk, of the assets" in the reference portfolio (Article 77d of the Directive 2009/138/EC);
  - c. **the risk-corrected currency spread, noted  $S_{RC-currency}$** , which corresponds to the difference between the spread  $S$  and the risk correction  $RC$ .

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<sup>10</sup>Article 77d of the Directive 2009/138/EC and Article 49 of the Delegated Regulation refer only to a reference portfolio for each relevant currency and country. Nonetheless, Article 49 provides that "the portfolio is based on relevant indices". The reference portfolio being a portfolio of indices which shall be representative for the assets which insurance and reinsurance undertakings are invested in according to Article 77d, EIOPA needs to build first a representative portfolio of assets for the purpose of composing the reference portfolio of indices.



146. In accordance with Article 50 of the Delegated Regulation, the spread  $S$  before risk correction is equal to the following:

$$S = w_{gov} \cdot \max(S_{gov}; 0) + w_{corp} \cdot \max(S_{corp}; 0)$$

where:

- a.  $w_{gov}$  denotes the ratio of the value of government bonds included in the reference portfolio of assets for that currency or country and the value of all the assets included in that reference portfolio;
- b.  $S_{gov}$  denotes the average currency spread on government bonds<sup>11</sup> included in the reference portfolio of assets for that currency or country;
- c.  $w_{corp}$  denotes the ratio of the value of bonds other than government bonds, loans and securitisations included in the reference portfolio of assets for that currency or country and the value of all the assets included in that reference portfolio;
- d.  $S_{corp}$  denotes the average currency spread on bonds other than government bonds, loans and securitisations included in the reference portfolio of assets for that currency or country.

147. The risk correction  $RC$  is equal to the following:

$$RC = w_{gov} \cdot \max(RC_{gov}, 0) + w_{corp} \cdot \max(RC_{corp}, 0)$$

where:

- a.  $w_{gov}$  denotes the ratio used for the calculation of the spread  $S$ ;
- b.  $RC_{gov}$  denotes the risk correction corresponding to the portion of the spread  $S_{gov}$  that is attributable to a realistic assessment of the expected losses, unexpected credit risk or any other risk<sup>12</sup>;
- c.  $w_{corp}$  denotes the ratio used for the calculation of the spread  $S$ ;
- d.  $RC_{corp}$  denotes the risk correction corresponding to the portion of the spread  $S_{corp}$  that is attributable to a realistic assessment of the expected losses, unexpected credit risk or any other risk.

148. The risk-corrected currency spread  $S_{RC-currency}$  is equal to the following:

$$S_{RC-currency} = S - RC$$

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<sup>11</sup> For the purpose of the calculation of the volatility adjustment, 'government bonds' means exposures to central governments and central banks.

<sup>12</sup> According to Article 51 of the Delegated Regulation, the risk correction shall be calculated in the same manner as the fundamental spread for a matching adjustment referred to in Article 77c of the Directive.

149. The risk-corrected currency spread may be negative (when  $RC > S$ ), since the zero floor mentioned in Article 50 of the Delegated Regulation only applies at portfolio level to the spread before the risk correction.

150. For each relevant currency, the currency volatility adjustment is equal to the following:

$$VA_{crncy} = 0.65 * S_{RC-crncy}$$

151. Therefore the currency volatility adjustment may be negative. The following table summarizes the application of floors in the process of calculation of the currency volatility adjustment:

	Market spread	Risk correction	Risk-corrected spread
For each individual bond	No floor (it may be either positive or negative)	the risk correction cannot be negative for each individual bond, (and hence at portfolio level as well)	Difference market spread – risk correction with no floor (i.e. it may be negative)
At portfolio level	Market spread at portfolio level cannot be negative		Difference market spread – risk correction with no floor (i.e. it may be negative)

#### 7.A.2. Country specific increase of the volatility adjustment

152. For each relevant country, the currency volatility adjustment shall be increased by the difference between the risk-corrected country spread  $S_{RC-country}$  and twice the risk-corrected currency spread, whenever that difference is positive and the risk-corrected country spread is higher than 100 basis points.

153. In order to determine a country specific increase of the volatility adjustment, EIOPA uses the following inputs:

- a. A **country representative portfolio** of bonds, securitisations, loans (including mortgage loans), equity and property covering the best estimate of obligations sold in that country, based on insurance market data collected by the means of the regulatory reporting;
- b. A **country reference portfolio** of indices based on the aforementioned representative portfolio.

154. Those inputs are used to calculate the following outputs:

- a. **the country spread S** between the interest rate derived from the reference portfolio of indices and the rates of the relevant basic risk-free interest rate term structure;

- b. **the portion of the country spread S, noted RC for risk correction**, which shall correspond to “the portion of the spread that is attributable to a realistic assessment of expected losses, unexpected credit risk or any other risk, of the assets” in the reference portfolio (Article 77d of the Directive 2009/138/EC);
  - c. **the risk-corrected country spread, noted  $S_{RC-country}$** , which corresponds to the difference between the spread  $S$  and the risk correction  $RC$ .
155. The country spread  $S$ , the risk correction  $RC$  and the risk-corrected country spread  $S_{RC-country}$  are calculated in the same way as the currency spread  $S$ , the risk correction  $RC$  and the risk-corrected spread  $S_{RC-crcny}$  for the currency of that country, but based on the inputs stemming from the country representative portfolio and the country reference portfolio.
156. For each relevant country, a country specific increase of the volatility may also apply, in such a manner that the total volatility adjustment is equal to the following:

$$VA_{total} = 0.65 * (S_{RC-crcny} + \max(S_{RC-country} - 2.S_{RC-crcny}; 0))$$

With  $S_{RC-country} > 100$  basis points.

157. Where  $S_{RC-country}$  is lower than 100 basis points, there is no country specific increase of the volatility adjustment.

### 7.A.3. **Publication of the volatility adjustment**

158. According to Article 77d of Directive 2009/134/EC, the volatility adjustment is not an entity-specific adjustment. Its value should be the same for all the insurance or reinsurance obligations expressed in the same currency (unless the country specific increase applies).
159. There is not a volatility adjustment at group level. The influence of the volatility adjustment at group level will be derived from the volatility adjustment applied by each component of the group, according to the method of calculation of the group financial and solvency condition.

## **7.B. Conceptual framework of the matching adjustment**

160. The matching adjustment is an adjustment to the basic risk-free interest rate, based on the spread on an undertaking's own assigned portfolio of matching assets, less a Fundamental Spread that allows for default and downgrade risk.
161. Undertakings must calculate the Matching Adjustment themselves, based on their own assigned portfolios of eligible assets. Rather than publishing the Matching Adjustment, EIOPA publishes only the Fundamental Spreads that undertakings should use, decomposing:
  - a. the probability of default (PD) to use in the de-risking of the cash flows of the assigned assets,
  - b. the cost of downgrade (CoD), and
  - c. the long-term average spread (LTAS).
162. EIOPA does not publish the values of the matching adjustment, because this adjustment is entity-specific. EIOPA publishes both the probability of default and cost of downgrade for each relevant asset class, duration and credit quality step. The probability of default is published both in basis points (used for the calculation of the volatility adjustment) and in percentages (used to de-risk cash flows in the matching adjustment). The cost of downgrade is published in basis points.
163. The steps involved in calculating the Matching Adjustment are set out in Article 77c of Directive 2009/138/EC and Articles [52 to 54] of the Delegated Regulation.
164. For each relevant currency, the Matching Adjustment for an undertaking will be a single number expressed in basis points. This single number should be added to the basic risk-free interest rate term structure for that currency at all maturities (i.e. it should be applied as a parallel shift of the whole of the basic risk-free curve).

## **8. Deriving the representative portfolios of bonds and the reference portfolios of '*yield market indices*' for the Volatility Adjustment**

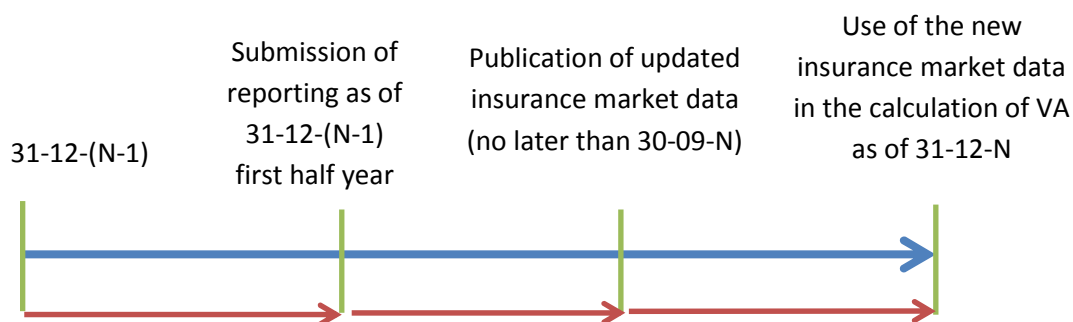
### **8.A. Introduction**

165. The organization of this section follows the conceptual framework described in the previous section.
166. This section analyses firstly the relationship among the representative portfolios applied for the currency VA and the country specific increase of the VA.
167. Secondly, the calculation of the representative portfolio of central government and central bank bonds, and the representative portfolio of other assets set out in Article 50 of the Delegated Regulation, are detailed.
168. Thirdly the calculation of the weights set out in Article [50] of the Delegated Regulation is described
169. Finally the calculation of the reference portfolios of '*yield market indices*' is disclosed for the representative portfolio of central government and central bank bonds, and the representative portfolio of other assets set out in Article 50 of the Delegated Regulation.
170. While the conceptual and process content of this section refers to the publication of the technical information once Solvency II is into force, the concrete insurance market data of this section refers solely to the publication of the technical information set out in Article 77e during the preparatory phase.
171. For the purpose of the preparatory phase in 2015, the data collected to build the representative portfolios are taken from the EIOPA Stress Test 2014 exercise<sup>13</sup>. The reference date for this exercise was 31/12/2013. In Annex to this section there is a more detailed description of the process applied to EIOPA Stress Test information in order to derive the relevant data.
172. For the first implementation of Solvency II in 2016, the data collected to build the representative portfolios will be taken from the preparatory reporting templates submitted by the participating insurance and reinsurance undertakings in 2015 to their respective National Competent Authority (NCA). The reference date for this exercise is 31/12/2014.

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<sup>13</sup> Excel file Stress Test 2014 Reporting Template (worksheets "BS" and "BS+")

173. Since 1/1/2016, EIOPA will update on annual basis the insurance market data, based on the annual regulatory reporting of undertakings. The insurance market data referred to year end N-1, (which undertakings will report in year N) will be used for the calculation of the technical information that undertakings should apply with reference to their situation at the end of year N. Updated insurance market data will be published at least three months before the year end N.



**8.B. Introductory remarks on the representative portfolios applied in the calculation of the currency volatility adjustment and in the calculation of the country specific increase of the volatility adjustment.**

174. According to Article 77d of the Directive 2009/138/EC, the currency volatility adjustment shall be based on a reference portfolio *“representative for the assets which are denominated in that currency and which insurance and reinsurance undertakings are invested in to cover the best estimate for insurance and reinsurance obligations denominated in that currency”*.
175. According to the same Article, the country specific increase of the volatility adjustment shall be based on a reference portfolio *“representative for the assets which insurance and reinsurance undertakings are invested in to cover the best estimate for insurance and reinsurance obligations sold in the insurance market of that country and denominated in the currency of that country”*.
176. Therefore, the scope of assets to include in the currency and country representative portfolios is different. However, in the Solvency II framework, insurance and reinsurance undertakings are not required to identify the assets covering their best estimate (except in the case of those covering insurance and reinsurance obligations applying the matching adjustment or under a ring fenced fund regime). It is not required either to identify the assets covering the best estimate of the

insurance or reinsurance obligations according to the country where they are sold.

177. In order to implement Article 77d of Directive 2009/138/EC in the simplest possible manner EIOPA applies the following proxies:
- a. For the currency representative portfolio: A calculation considering that all assets in a currency X cover liabilities in currency X. Hence, the currency representative portfolio of currency X is based on all assets denominated in that currency X and in which undertakings are invested in<sup>14</sup>.
  - b. For the country representative portfolio: A calculation considering that all liabilities are sold in the country of the undertaking and denominated in the currency of that country. Hence, the country representative portfolio of country A is based on all assets in which undertakings established in that particular country are invested in.
178. These assumptions will be monitored in the future and also they may be removed when there is evidence to the contrary (e.g. for a certain market). The evidence used to remove either or both of these assumptions will be centrally validated by EIOPA.
179. The calculation of the two different sets of reference portfolios (currency VA and country specific increase of the VA, respectively) is feasible for the EEA currencies, since the information contained in the individual reporting at solo level provides the data necessary for the purpose.
180. In the case of non-EEA currencies, the information contained in the reporting at group level allows a proxy only for the calculation of the currency volatility adjustment. Therefore for non-EEA currencies, the only feasible approach is to apply the portfolios used for the calculation of the currency adjustment also for the country specific increase of the volatility adjustment.

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<sup>14</sup> Therefore, the representative portfolio for a currency X may include as issuer country Y with a different currency, when country Y issued bonds expressed in currency X and hold by undertakings in country X.

### 8.C. Representative portfolios of assets referred to in Article 50 of the Delegated Regulation

181. During the process of collection of data, the real bonds (i.e. the representative portfolio of bonds) should be mapped to a given granularity of '*yield market indices*' (i.e. the representative portfolio of assets is transformed in the reference portfolio of yield market indices). The expression '*yield market indices*' covers in this section both yield curves and indices on yields
182. The representative portfolio for each relevant currency and each relevant country allows to collect the following information:
- The **market value** of the assets included in the representative portfolio. Those market values are required to calculate the weights  $w_{gov}$  and  $w_{corp}$  and the risk-corrected spread  $S_{RC}$ .
  - The **duration** of the bonds, loans and securitizations included in the representative portfolio. Those durations are required to make the spread  $S$  maturity-dependent and to select the relevant yield market indices.
  - The **credit quality step** of the bonds, loans and securitizations included in the representative portfolio. Those credit quality steps are required to calculate the spread  $S$  and the risk correction  $RC$  and to select the relevant yield market indices.
  - The **asset class**, understood as economic sector, of the bonds other than government bonds, loans and securitizations included in the representative portfolio. Those asset classes are required to select the relevant yield market indices.
183. As a general rule averaging the three dimensions mentioned in (b), (c) and (d) is not relevant since neither the spreads nor the PD and CoD behave linearly (in fact in most of cases they behave really far from a linear average).
184. The purpose of collecting such data is twofold:
- Calculating the weights  $w_{gov}$  and  $w_{corp}$  set out in Article [50] of the Delegated Regulation; and
  - Mapping the bonds, loans and securitizations with the relevant market yield indices.
185. On the basis of the information collected, EIOPA determines for the insurance and reinsurance market of each country, the aggregated market value and the weighted average duration. For this purpose, the aggregated market values are simply summed up whereas, for durations, the average is weighted by the market value of each bond.



186. For central government and central bank bonds, EIOPA determines the aggregated market value and the weighted average duration by issuers. This also applies to non-central government bonds<sup>15</sup>, although they are assimilated to bonds other than government bonds by virtue of Article 50 of the Delegated Regulation.
187. For corporate bonds, loans and securitisations, EIOPA determines the aggregated market value and the weighted average duration according to the following two dimensions:
- a. Asset classes: EIOPA applies the following granularity: 'Financial assets', 'Non financial assets', 'Securitizations', 'Loans' and 'Non-central government bonds';
  - b. Credit quality steps: the market value of bonds other than government bonds, loans and securitizations is split and aggregated by bucket of credit quality step (CQS) from 0 to 6.  
  
Unrated assets will be distributed into CQS proportionally to the composition of the rated assets. EIOPA will monitor whether the criteria above is providing wrong incentives regarding the assessment by undertakings of the credit quality of their assets.
188. As mentioned above, for EEA currencies, the process described in this section shall be based on individual reporting at solo level. For non-EEA currencies, the process shall be based on the reporting at group level.

#### **8.D. Calculation of the weights set out in Article 50 of the Delegated Regulation**

189. This subsection describes the procedure currently applied. As soon as more detailed information become available, the process will evolve.
190. The following table reflects the items composing the total representative portfolio described in Article [49] of the Delegated Regulation and the items considered as part of the two portfolios referred to in Article [50] of the Delegated Regulation. The table below is based on the balance sheet contained in the baseline of EIOPA Stress Test 2014<sup>16</sup>.

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<sup>15</sup> At least for the preparatory phase.

<sup>16</sup>EIOPA has decided that Stress Test 2014 data are the best current basis for the calculation of the weights. As would be the case with any data set selected, the resulting weights are dependent on

191. (G) identifies the items considered in the portfolio of central government and central bank bonds, (C) the items considered in the portfolio of bonds other than government bonds, loans and securitisations, and (N) the rest of items belonging to the representative portfolio.
192. Investments of insurance and reinsurance undertakings in collective investment undertakings and other investment packaged as funds are treated as investments in the underlying assets.
193. Assets assigned to a matching adjustment portfolio should be excluded from the representative portfolio.
194. Assets covering unit and index linked business should be considered in the calculation of the weights and of the representative portfolios of the VA, unless they cover technical provisions calculated as a whole.
195. For the preparatory phase and in absence of information on the composition of these assets, EIOPA will assume that the composition of the assets covering the unit and index-linked business is sufficiently similar to the composition of the rest of assets considered for the purpose of the VA. This assumption will be removed once EIOPA receives adequate information on the composition of the assets covering the unit and index-linked portfolios of insurance and reinsurance obligations.
196. The following table shows the items of the balance sheet considered in the calculation of the weights set out in Article [50] of the Delegated Regulation.

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the characteristics of the participants, for example in this case in the extent of use of the matching adjustment.

Property (other than for own use)	<b>N</b>
Participations	<b>N</b>
Equities	
Equities – listed	<b>N</b>
Equities – unlisted	<b>N</b>
Bonds	
<b>Government Bonds</b>	<b>G</b>
<b>Corporate Bonds</b>	<b>C</b>
<b>Structured notes</b>	<b>C</b>
<b>Collateralised securities</b>	<b>C</b>
Investment funds	
<i>Equity funds</i>	<b>N</b>
<i>Debt funds</i>	<b>C</b>
<i>Real estate funds</i>	<b>N</b>
<i>Alternative funds</i>	<b>N</b>
<i>Private equity funds</i>	<b>N</b>
Loans & mortgages	
<i>Loans &amp; mortgages to individuals</i>	<b>C</b>
<i>Other loans &amp; mortgages</i>	<b>C</b>
<i>Loans on policies</i>	<b>C</b>

197. The weights  $w_{gov}$  and  $w_{corp}$  referred to in Article [50] of the Delegated Regulation of this Technical documentation are calculated according to the following formulas<sup>17</sup>:

$$w_{gov} = \frac{MV_1 - portion_{gov} \cdot MV_{MA}}{MV_1 + MV_2 + MV_3 + MV_4 + MV_5 + MV_6 - MV_{MA}}$$

$$w_{corp} = \frac{MV_2 + MV_3 + MV_4 - portion_{corp} \cdot MV_{MA}}{MV_1 + MV_2 + MV_3 + MV_4 + MV_5 + MV_6 - MV_{MA}}$$

where  $MV$  denotes the aggregated market values of the assets considered in the subscripted category:

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<sup>17</sup> In light of the information available, for the preparatory phase  $MV_{MA}$  has been approximated with the value of the technical provisions where the matching adjustment applies.

1=central government and central bank bonds ;

2=corporate bonds ;

3=loans ;

4=securitisations ;

5=equity ;

6=property ;

*MA*=assets covering the portfolios of insurance and reinsurance obligations where the matching adjustment applies.

We have  $gov=\{1\}$  and  $corp=\{2,3,4\}$  and there holds:

$$MV_{MA} \leq MV_1 + MV_2 + MV_3 + MV_4$$

198. The factors  $portion_{gov}$  and  $portion_{corp}$  are calculated as:

$$portion_{gov} = \frac{MV_1}{MV_1 + MV_2 + MV_3 + MV_4}$$

$$portion_{corp} = \frac{MV_2 + MV_3 + MV_4}{MV_1 + MV_2 + MV_3 + MV_4} = 1 - portion_{gov}$$

199. The weights  $w_{gov}$  and  $w_{corp}$  for the preparatory phase in 2015 and for EEA currencies are the following ones:

**Table 8. EEA countries. Weights set out in Article 50 of the Delegated Regulation applied for the technical information set out in Article 77e(1c) of Directive 2009/138/EC**

	Weights Govts	Weights Corps		Weights Govts	Weights Corps
Euro	38.7%	48.2%			
AT	31.6%	46.4%	IT	62.0%	25.1%
BE	55.3%	34.3%	LV	89.7%	9.4%
BG	54.4%	23.8%	LT	69.1%	25.7%
CR	74.3%	9.4%	LU	40.2%	49.8%
CY	20.2%	63.9%	MT	49.7%	35.7%
CZ	52.6%	29.2%	NL	41.1%	48.5%
DK	19.3%	61.9%	NO	18.4%	53.8%
EE	64.0%	27.3%	PL	44.8%	11.8%
FI	18.0%	46.0%	PT	47.1%	44.8%
FR	37.1%	47.6%	RO	74.5%	12.2%
DE	22.4%	68.5%	SK	53.6%	39.8%
GR	54.4%	35.3%	SI	46.0%	38.1%
HU	83.4%	9.7%	ES	50.1%	37.7%
IE	40.9%	52.5%	SE	20.6%	31.6%
			UK	16.7%	30.3%

200. For Liechtenstein the VA for the Swiss Franck will apply.
201. For Iceland the VA will be zero because there is no reliable information on the inputs necessary for its calculation.
202. The last subsection in item 8. describes the approach for non-EEA currencies during the preparatory phase.

## 8.E. Reference portfolios of 'yield market indices'

203. In order to be compliant with Articles 77b, 77c and 77d of Directive 2009/138/EC, the definition of the reference portfolios of '*yield market indices*' needs to be granular enough to reflect the duration, credit quality and asset class of the 'yield market indices'. This is critical to ascertain an appropriate calibration of the volatility adjustment and the matching adjustment because the spread, the risk correction and the fundamental spread depend to a great extent on those features. Furthermore, such dependence is not linear and therefore the use of simple averages or baskets materially deviates from the relevant calculation
204. EIOPA uses a reference portfolio for each relevant currency and country to calculate the volatility and matching adjustment according to the following information:
- a. **Data from the relevant government bonds yield market indices.** Those data are required to determine the interest rates of government bonds including in the representative portfolio, by duration and country of issuance. Those interest rates are then used to compute the spread  $S$  and the risk correction  $RC$  for government bonds.
  - b. **Data from the relevant corporate bonds yield market indices.** Those data are required to determine the interest rates of corporate bonds including in the representative portfolio, by duration, asset class and credit quality step. Those interest rates are then used to compute the spread  $S$  and the risk correction  $RC$  for corporate bonds.
  - c. Currently EIOPA does not use market data to derive the spread  $S$  and the risk correction  $RC$  for **loans and securitisations included in the representative portfolios**. The assumption underlying this choice is that the spread  $S$  and the risk correction  $RC$  for loans and for securitizations are sufficiently similar to those for corporate bonds with the same credit quality and duration. EIOPA will test this assumption and may remove it in the future to the extent that there are appropriate indices for loans and for securitisations, which are readily available to the public and for which there are published criteria for when and how the constituents of those indices will be changed, in accordance with Article [49] of the Delegated Regulation.
205. The currency and country reference portfolios are built on the basis of the representative portfolios of the same currency or country. For this purpose, a mapping is made to associate the characteristics of the assets including in the representative portfolios with indices.

For central governments and central banks bonds. Currency portfolio

206. The reference portfolio of '*yield market indices*' used to calculate the VA for a given currency has as many model bonds as central government and central banks issuing bonds in that currency (and which insurance and reinsurance undertakings are invested in).
207. The calculations for each issuer are based on its specific yield curve ('*yield market index*') according to the average duration, at the currency area level, of the issuances. Linear interpolation is used to derive the interannual rates corresponding to the average duration.
208. For the sake of simplicity, exposures are expressed in percentages and rounded to the nearest percentage.<sup>18</sup>
209. In the case of the Euro area, all the issuers of the Euro zone are mapped with a single '*yield market index*': the relevant maturity of the ECB curve for all government bonds of the Euro zone (daily observations of annual spot rates). EIOPA provides the necessary information to allow the reconstruction of the LTAS of this curve.

For central governments and central banks bonds. Portfolio for the country specific increase of the volatility adjustment

210. For each '*country reference portfolio*', EIOPA selects as many '*yield market indices*' as issuers of government bonds and central bank bonds in which undertakings of that country are invested in. The market yield for each issuer is derived from the government bond yield curves listed in subsection 3.C, according to the relevant duration. Linear interpolation is used to derive the interannual rates corresponding to that duration.
211. In case of issuances in a currency different than the currency of the issuer, the use of the yield curve in the currency of the issuer is considered to be an acceptable proxy.
212. Using yield curves allows EIOPA to collect interest rates of government bonds for several maturities. Furthermore, the yield curves should be consistent with those used for the calculation of the basic risk-free interest rates term structures in the case of currencies without DLT swaps.
213. For the sake of simplicity, exposures are expressed in percentages and rounded to the nearest percentage as for the currency portfolio.

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<sup>18</sup> In case the total exposure after rounding is not 100%, the rounding differences (positive or negative) are allocated to the largest exposure.

214. In case there is no government yield curve for a country of the Euro zone , EIOPA applies the following criteria:

the national increase of the VA will be zero,

the long term average spread of the government bonds will be approximated with the long term average spread of a peer country, considering those countries with similar credit quality and level of interest rates for the financial instruments used for the respective basic risk-free curves.

**Table 9. Peer countries as issuers for the calculation of the long term average spreads of government bonds**

Country without govts. yield curve	Peer country
Cyprus	Greece
Estonia	Belgium
Latvia	Spain
Liechtenstein	Switzerland
Lithuania	Spain
Luxemburg	France
Malta	Ireland

For corporate bonds.

215. Regarding corporate bonds, further than the duration, the following dimensions are considered:

- a. Assets classes, with a differentiation among 'financial' and 'non financial exposures',
- b. Credit quality steps as set out in the Delegated Regulation (from 0 to 6),
- c. Currencies, with a differentiation where possible for the Euro, GBP and USD.

216. Section 11 lists the market yield indices used for the implementation of this granularity.



217. Exposures are expressed in percentages and rounded to the nearest percentage<sup>19</sup>. Therefore the theoretical 42 model corporate bonds resulting from the granularity mentioned above, in practice and for most of markets, is limited to just a few market yield indices.
218. The following table reflects the allocation of the ratings used by the market providers to credit quality steps for the only purposes of this technical documentation. EIOPA states explicitly that this allocation does not pre-empt the work in progress regarding the ratings of ECAIs in relation with the Delegated Regulation

**Table 10. Allocation of ratings to credit quality steps  
(only for the purpose of the technical information set out in  
Article 77e of Directive 2009/138/EC)**

iBoxx or S&P rating	CQS	iBoxx or S&P rating	CQS
AAA	0	BB	4
AA	1	B	5
A	2	CCC	6
BBB	3	CC, C,...	6

219. Non-central government bonds included in the portfolio of '*assets other than central government bond and central banks bonds*', are not split by economic sectors and credit quality steps. Their yields and durations are those specific of the issuer in the portfolio of '*assets other than central government bond and central banks bonds*' (i.e. the approach for central governments and central banks bonds of the Euro area is not applied to non-central governments bonds)s.
220. For the time being and due to the lack of data, no specific model bonds have been developed specifically for securitizations and loans. Once the relevant information is available, it will be necessary to assess the impact on the number of model points of a specific consideration of securitizations and loans (including mortgage loans).

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<sup>19</sup> In case the total exposure after rounding is not 100%, the rounding differences (positive or negative) are allocated to the largest exposure.

## 8.F. Volatility Adjustment for non-EEA currencies

221. Due to the incompleteness of the available information, EIOPA has carried out an ad hoc survey based on market data at group level regarding exposures denominated in five non-EEA currencies: Australian Dollar, Canadian Dollar, Swiss Franc, Japanese Yen and US Dollar. The selection of these currencies was based on the information available.
222. The information on non-EEA portfolios has material limitations compared to the information of the EIOPA Stress Test 2014. EIOPA highlights the possibility of variations in the outputs, once a better set of information becomes available. The weights that EIOPA will apply during the preparatory phase are the following ones:

**Table 11. non-EEA countries. Weights set out in Article 50 of the Delegated Regulation**

	Central govts.	Others
Australia	76.5%	18.2%
Canada	51.9%	41.1%
Switzerland	23.8%	51.4%
Japan	85.2%	11.4%
USA	18.2%	76.1%

223. EIOPA will assess during 2015 the relevance of publishing the volatility adjustment for other non-EEA currencies on a case by case and considering, among other factors, the materiality of the currency both at the individual and market level.

## 9. Methodology for the determination of the risk corrections and the fundamental spreads

### 9.A. Introduction

224. In this section the expression '*risk correction*' refers to the volatility adjustment. The expression '*fundamental spread*' refers to the matching adjustment. The expression '*sovereign debt*' refers to both '*central governments and central banks bonds*' and also to other government bonds.

225. Article [51]of Delegated Regulation specifies that the risk correction “shall be calculated in the same manner as the fundamental spread” and using the same inputs. Therefore, the methods and source data described in this section are relevant for both the risk correction used for the volatility adjustment and the fundamental spread applied for the matching adjustment.
226. In the absence of specific reference to the contrary, the content of this section refers to both the risk correction spread and the fundamental spread.

### **9.B. Determination of the risk-corrections and the fundamental spreads for central government and central bank bonds**

227. According to Article 77c of the Directive 2009/138/EC, the fundamental spread on government bonds is equal to the maximum between:
- a. The sum of the credit spread corresponding to the probability of default of the assets considered and the credit spread corresponding to the expected loss resulting from downgrading of the assets concerned;
  - b. A percentage of the long-term average of the spread, over the basic risk-free interest rate, of assets of the same duration, credit quality and asset class, as observed in financial markets.
- This percentage is 30% for exposures to Member States' central governments and central banks, and 35% for exposures to non Member States' central governments and central banks (Article 77c(2)(b) and (c) of Directive 2009/138/EC).
228. Recital 22 of the Delegated Regulation specifies that *‘where no reliable credit spread can be derived from the default statistics, as in the case of exposures to sovereign debt, the fundamental spread for the calculation of the matching adjustment and the volatility adjustment should be equal to the long-term average of the spread over the risk-free interest rate set out in Article 77c(2)(b) and (c) of Directive 2009/138/EC’*.
229. Therefore, the risk correction of the spread  $S_{gov}$  and the fundamental spread on central government bonds corresponds only to:
- $$RC = FS = 30\% \text{ LTAS for exposures to Member States' (of the European Union) central governments and central banks}$$
- $$RC = FS = 35\% \text{ LTAS for exposures to non Member States' central governments and central banks}$$

where LTAS is the long-term average of the spread over the risk-free interest rate of assets of the same duration, credit quality and asset class.

9.B.1. **Long-term average of the spread on central government and central bank bonds**

230. Article 54(3) of the Delegated Regulation provides the following:
- a. The long-term average shall be based on data referring to the last 30 years;
  - b. Where a part of that data is not available, it shall be replaced by constructed data;
  - c. The constructed data shall be based on the available and reliable data referring to the last 30 years. Data that are not reliable shall be replaced by constructed data using that methodology;
  - d. The constructed data shall be based on prudent assumptions.
231. In order to determine the long-term average for each relevant currency and country, EIOPA needs the following inputs:
- a. The zero-coupon yield curve of the government bonds in the government bonds representative portfolio, over the last 30 years;
  - b. The basic risk-free interest rate term structure denominated in the currency of the bonds in the government bonds representative portfolio, over the last 30 years.
232. However, in most cases there is no historical data over a 30 years period on interest rate swaps and government bonds.
233. To overcome this issue, EIOPA re-constructs missing data, in accordance with Article 54(3) of the Delegated Regulation, applying the following rule: the missing spread data for each currency and maturity are re-constructed using the average spread calculated with the data available from 1/1/1985 or, failing that, whenever reliable spread data are available.
234. Nevertheless, since the overnight market have developed only since the end of the last century, the availability of overnight swap rates (necessary to calculate the credit risk adjustment) has been limited, resulting de facto in a calculation of the LTAS since 1/1/1999 for all currencies.
235. Therefore, EIOPA assumes the average spread over the period for which data are missing is not materially different from the average spread that can be calculated with available data.
236. To illustrate the implementation of this rule, let's take the following example. Suppose that the volatility adjustment is calculated at year end 2015. Suppose further that, for a given currency and maturity, data are only available from 01/01/1999 till 31/12/2015 (i.e. 17 years). The assumption is that the constructed data have the same average as the average obtained from the available market data:

- a. From 1986 to 1998: the constructed spread for each year corresponds to the flat average spread calculated on the period 1999-2015.
  - b. From 1999 to 2015: the available spread data are used.
237. EIOPA will determine the constructed spread for each currency and maturity where data are missing on the basis of the data available at 31/12/2015. All the calculations are developed using daily data.
238. From 1/1/2016 until having the complete 30-years historical series from January 1999, at each publication the LTAS will be calculated as:

$$\frac{LTAS_{31_12_2015} * (7800 - ntd) + \sum^{ntd} Spreads\_from\_1_1_2016}{7800}$$

where *ntd* denotes the number of new trading days from 1/1/2016;  $\sum^{ntd} Spreads\_from\_1_1_2016$  means the sum of the spreads during those new dates;  $LTAS_{31_12_2015}$  identifies the LTAS as of 31/12/2015; and it is assumed that a 30 years period is composed of 7800 trading days.

239. For the sake of transparency EIOPA will publish the long-term average spreads.
240. The calculations according to the methodology above show that for most of currencies, the markets of government bonds with more than 10 years duration have developed only from the first half of the last decade. As a consequence, the calculation of the LTAS for maturities higher than 10 years lacks of representativeness due to the reduced number of observations and to the fact that a major part of the observations refer to the current financial crisis.
241. In order to avoid this bias, the calculation of LTAS for government bonds is carried out from 1 to 10 year maturities. The LTAS resulting for maturity 10 years is applied for longer maturities. Even below 10 years, for a few currencies some maturities deliver non plausible results. The following table reflects the currencies with some maturity delivering non plausible LTAS.

**Table 12. LTAS for disregarded maturities for LTAS central governments**  
( 0 = disregarded and then interpolated ; 1 = LTAS historical data)

Country	ISO 4217	1	2	3	4	5	6	7	8	9	10
Austria	AT	1	1	1	0	1	1	1	1	1	1
Croatia	HRK	0	1	1	1	1	0	1	1	1	1
Cyprus	CY	1	1	1	1	1	0	1	1	1	1
Czech Republic	CZK	1	1	1	1	1	1	1	1	0	1
Denmark	DKK	1	1	1	1	1	1	1	1	0	1
Greece	GR	1	1	0	1	1	0	1	1	1	1
Hungary	HUF	0	0	1	1	1	1	1	1	1	1
Norway	NOK	1	1	1	1	1	1	1	1	1	0
Romania	RON	1	1	1	1	1	0	1	1	1	1
Russia	RUB	1	1	1	1	1	1	1	1	1	0
Slovakia	SK	1	1	1	0	1	1	1	1	1	1
Slovenia	SI	1	1	0	1	1	1	1	1	1	1
Sweden	SEK	1	1	1	1	1	1	1	1	0	1
Switzerland	CHF	0	1	1	1	1	1	1	1	0	1
Chile	CLP	1	1	1	1	1	1	1	0	1	1
Malaysia	MYR	1	1	1	1	1	1	1	1	1	0
Korea, South	KRW	1	1	1	1	1	0	1	0	0	0
Thailand	THB	1	1	1	1	1	0	1	1	1	1

### 9.C. Determination of the risk-corrections and fundamental spreads for assets other than central government and central bank bonds

#### 9.C.1. General elements

242. Directive 2009/138/EC and Articles [49 to 54] of the Delegated Regulation set down several aspects of the methodology for calculating the Risk Correction and the Fundamental Spread of assets other than central government and central bank bonds. The methodology to be used is different depending on whether reliable credit spreads can be determined from long-term default statistics.

243. Where reliable credit spreads *can* be derived from such statistics, the risk correction spread and the fundamental spread can be expressed as:

$$RC = FS = \text{MAX} ( PD + \text{CoD}, 35\% \text{ LTAS} ) \text{ where}$$

PD = the credit spread corresponding to the probability of default on the assets;

CoD = the credit spread corresponding to the expected loss resulting from downgrading of the assets;

LTAS = the long-term average of the spread over the risk-free interest rate of assets of the same duration, credit quality and asset class.

244. Where no reliable credit spreads can be derived from long-term default statistics, the risk correction and fundamental spread can be expressed as  $RC = FS = 35\% LTAS$ , where LTAS is the long-term average of the spread over the risk-free interest rate of assets of the same duration, credit quality and asset class.
245. The Delegated Regulation set the recovery rate assumption in the event of a default at 30% for all asset classes.
246. The Delegated Regulation also specify that the LTAS should be based on data of the last 30 years.
247. Where there is not 30 years of complete and reliable information relating to spreads, the Delegated Regulation specify that the 'missing' data should be constructed using the data that is available, in a prudent manner. The process of reconstruction is consistent with the process described above for sovereign bonds.
248. Where the fundamental spread is defined by the 35% LTAS, the difference among the fundamental spread and the PD will be attributed to the CoD.

#### 9.C.2. **Method for deriving the probability of default (PD) and the cost of downgrade (CoD)**

249. The calculation of the PD derives an amount that is interpreted as an investor's required compensation for assuming the risk of the expected probability of default of a bond. The expectation of a default (based on historical default probabilities derived from the transition matrices) is thus combined with an assumption on the recovery value in case of default, which is assumed to be 30% of the market value as set out in Article [54 (2)] of the Delegated Regulation.
250. For the sake of consistency, EIOPA applies the same method to calculate both the PD and CoD with only a slight difference. The hold-to-maturity view for matching portfolios reduces the importance of the rebalancing requirement underlying the CoD computation (Art. 54 (4) of the Delegated Regulation). Therefore, the CoD spread computation uses the transition matrix adjusted for cost accounting, while the PD spread computation uses the unadjusted transition matrix. Both, the PD and CoD components, are still based on the same inputs:

empirical one-year transition matrices, the relevant basic risk-free interest rates term structure and for each credit quality step a vector of relevant portions of the market value of a risk-free benchmark instrument. These portions have been designed to be analogous to the recovery rate for the PD.

**Table 13. Vector of scaling factors used in the calculation of the Cost of Downgrade**

CQS	Rc	CQS	Rc
AAA	98%	BB	70%
AA	97%	B	50%
A	95%	CCC	40%
BBB	85%		

251. In case of a rating migration to a credit quality step of lower quality (downgrades), the cost is defined as difference between the two market values. This cost reflects the cost of replacing the downgraded asset with an asset of the same credit quality it was downgraded from and preserving the original cash flow pattern.
252. For the next year of projection the asset is supposed to start from the credit quality step of the replaced bond. This cost accounting and rebalancing procedure is applied until maturity of the original bond. This procedure implements the rebalancing requirement as set out in Article [54 (4)] of the Delegated Regulation.
253. The total loss is defined as the loss in market value by subtracting the present value of future downgrading cost cash flows. Finally, the loss in market value is transformed into an implied (higher) yield and the result is expressed as spread over the basic risk free interest rate in basis points.
254. Annex to this subsection contains a detailed description of this method. Subsection 11.B.2. details the transition matrices used for the calculations described in this subsection.
255. For the calculation of the volatility adjustment, the value of the PD and CoD expressed in basis points are rounded to the nearest basis point. This rounded value is used as input in the relevant step of the calculation of the volatility adjustment.
256. For the matching adjustment, the PD that EIOPA publishes is the probability to apply for the de-risking of cash flows as follows:

$$\text{de-risked cash flow} = \text{cashflow} * (1 - PD_{EIOPA}) + \text{recovery\_rate} * \text{cashflow} * PD_{EIOPA}$$



257. The PD probability for de-risking cash flows expected at time 't' is derived from a Markov matrix as the last column obtained when powering 't' times the one year average transition matrices (see the annex for further details).
258. The probability of default, cost of downgrade and fundamental spread are published until 30 years maturity. From that maturity onwards the value of those magnitudes for the 30 years maturity will apply.

### 9.C.3. **Long-term average of the spread on other assets**

259. The long-term average of the spread on other assets is calculated in the same manner, *mutatis mutandis*, as the long-term average spread on government bonds described in the subsection 9.B.1 above, with the following specificities.
260. Having in mind the content of the market input data as described in section 11, the value of the 2 year LTAS is used also as value of the 1 year LTAS.
261. LTAS on other assets is kept constant from the last maturity available of the market source onwards.
262. For GBP Non financial bonds, credit quality step 1, the LTAS for maturities 4 to 8 years is obtained by linear interpolation of LTAS for 3 and 9 years maturities, because the history of the indices available in the range [4-8] does not allow a reliable calculation of those LTAS.

9.C.4. **Currencies without yield market indices for corporates, loans and securitizations.**

263. For currencies for which there are no yield market indices satisfying the calculation needs, the spread on corporate bonds denominated in Euro is used with an adjustment proportionate to the difference between the basic risk-free interest rate term structure of the concerned currency and the Euro. In such case, the following formulas apply:

$$S_{corp}^X = S_{corp}^{\epsilon} + \kappa \cdot (Y_{rfr}^X - Y_{rfr}^{\epsilon})$$

$$Y_{corp}^X = Y_{corp}^{\epsilon} + (1 + \kappa) \cdot (Y_{rfr}^X - Y_{rfr}^{\epsilon})$$

where  $\epsilon$  denotes the Euro,  $X$  refers to a currency without interest rates term structures for the assets relevant for the spread  $S_{corp}$ ,  $Y_{corp}$  denotes the yield of the respective corporate bonds of the same credit quality,  $Y_{rfr}$  denotes the basic risk free interest rate and  $\kappa=0.5$ . The inputs of this formula are maturity dependent according to the information available.

264. EIOPA may also consider the specific case of covered bonds, once the current limitations in the information available are solved.
265. For the time being an operational solution has been identified for the Danish market of covered bonds based on the following formula:

$$S_{covered}^{DKK} = R_{covered}^{DKK} - Y_{rfr}^{DKK}$$

where *DKK* denotes Danish Kroner and:

$R_{covered}^{DKK}$  shall be based on the yield from *Nykredits Realkreditindeks*. (Bloomberg ticker NYKDYTM)

The maturity used for  $Y_{rfr}^{DKK}$  shall correspond to the duration of the *Nykredits Realkreditindeks* (7 years).

266. The resulting  $S_{covered}^{DKK}$  is relevant for AAA Financials in the calculation for DKK.
267. *Nykredits realkreditindeks* includes a representative extract of the Danish covered bond market. The index includes both covered bonds with short and long maturities. See also the accompanying annex to this section.

#### 9.C.5. **Inputs used to determine $S_{gov}$ and $S_{corp}$**

268. For determining the spread  $S_{gov}$  on government bonds, the starting point is the information of insurance market data relevant for the currency (or country) whose VA is calculated. This information is composed of two elements:
- The composition of the reference portfolio of yield market indices of central government and central bank bonds for the currency (or country). This composition is applied considering for each component of the portfolio (i.e. each issuer) its relative market value (the percentage of the total market value of the portfolio).
  - It is also necessary to know the duration of each component of the reference portfolio.

Each relative market value and its corresponding duration build a model bond (i.e. a model bond is a government bond with the duration for such bond in the currency or country where the VA is calculated).

Since in the case of government bonds the selected yield market indices are yield curves, this means that each model bond is the value of the yield curve for each issuer at the relevant maturity.

269. The following financial market inputs are also necessary:
- The market yields corresponding to the currency and duration of each model point representing the government bonds as referred above and in section 8,
  - The basic risk-free interest rates corresponding to the currency and durations of each model point representing the government bonds as referred above and in section 8,
  - The risk corrections corresponding to the currency and durations of each model point representing the government bonds as referred above and in section 8.
270. Where the average duration of the relevant government bond in which the insurance and reinsurance undertakings of a given market are invested in does not coincide with one of the maturities of the yield curve, EIOPA uses a linear interpolation to find the interest rate of the government bond and/or the basic risk-free rate and/or the risk correction that corresponds to the average duration.
271. For determining the spread  $S_{corp}$  on assets other than central government and central bank bonds, the same approach applies *mutatis mutandis*.

## 10. Process of calculation of the risk-corrected spread at portfolio level

272. Process of calculation of the currency volatility adjustment (the process applies *mutatis mutandis* to the calculation of the country specific increase of the volatility adjustment).

Step 1.- For each currency, identify the model bonds (and their duration) included in the representative portfolio

Step 2.- For each model bond, input the market yield at the date of calculation, according to the table in section 11 and the duration of the model bond<sup>20</sup>. This yield is referred to in the process as '*yield before risk correction*'.

Step 3.- For each model bond, input the basic risk-free interest rates curve at the date of calculation, according to the duration of the model bond<sup>21</sup>.

Step 4.- For each model bond, calculate the risk correction as the maximum of the relevant percentage of the long-term average spread (30 or 35% as described in subsection 9.B), and the PD+CoD (probability of default and cost of downgrade, as referred to in subsection 9.C and its annex). In the case of sovereign debt, the risk correction is the relevant percentage of the long-term average spread (i.e. the PD+CoD component does not apply). Where the LTAS is negative, a zero floor is applied as mentioned in section 7.

Step 5.- Once completed the previous steps, a single cash flow is projected for each model bond according to the duration of the model bond, and using as capitalization rate the market '*yield before risk correction*' referred to in step 2. This means a cash flows projection with the features of each model bond.

Step 6.- The projection of single cash flows for each model bond made in step 5 is repeated but using as capitalization rate the basic risk-free rate referred to in step 3.

Step 7.- A third projection is necessary but using this time, as capitalization rate, the '*yield before risk correction*' reduced with the risk correction derived in step 4.

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<sup>20</sup>Where the market yield is given for a maturity that does not fit exactly the weighted average duration of the model bond, a linear interpolation of yields of the same index or the same curve is performed.

<sup>21</sup>The same linear interpolation as in step 2 applies if necessary.

Steps 8, 9 and 10.- Calculation of the three following internal effective rates (IER) for the overall reference portfolio:

- a. Step 8.- "IER\_yield\_before" is equal to the internal effective rate, calculated as a single discount rate that, where applied to the cash-flows calculated in step 5, results in a value that is equal to the aggregated value of the whole portfolio (since relative percentages are used, this aggregated value is 1);
- b. Step 9.- "IER\_basic\_RFR" is equal to the internal effective rate, calculated as a single discount rate that, where applied to the cash-flows calculated in step 6, results in a value that is equal to the aggregated value of the whole portfolio (since relative percentages are used, this aggregated value is 1);
- c. Step 10.- "IER\_yield\_corrected" is equal to the internal effective rate, calculated as a single discount rate that, where applied to the cash-flows calculated in step 7, results in a value that is equal to the aggregated value of the whole portfolio (since relative percentages are used, this aggregated value is 1).

273. Finally, for each relevant currency, the spreads  $S_{gov}$  (the same applies for  $S_{corp}$ ) before the risk correction is equal to the following, in accordance to Article 50 of the Delegated Regulation:

$$S_{gov} = \max(0; IER_{yield\ before\ RC} - IER_{BRFR})$$

while the risk correction  $RC_{gov}$  (the same applies to  $RC_{corp}$ ) is equal to the following<sup>22</sup>:

$$RC_{gov} = \max(0; IER_{yield\ before\ RC} - IER_{yield\ RC})$$

Finally, for each relevant currency and country the VA is calculated using these four values ( $S_{gov}$ ,  $S_{corp}$ ,  $RC_{gov}$ ,  $RC_{corp}$ ) as inputs to the formula referred to in subsection 7.A

274. The volatility adjustment is rounded at the nearest integer basis point. This rounding is applied only at the end of the calculation process.

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<sup>22</sup> The risk correction at portfolio level cannot be negative because, as mentioned in section 7, the risk correction for each individual model bond cannot be negative.

Illustrative example (dummy data)

Wgov	62,00%	
Wcorp	25,10%	
Sgov	0,85%	= IER 1(step 8) - IER 2 (step 9)
Scorp	1,20%	= IER 1(step 8) - IER 2 (step 9)
RC gov	0,20%	= IER 1(step 8) - IER 3 (step 10)
RC corp	0,35%	= IER 1(step 8) - IER 3 (step 10)
S	0,83%	
RC	0,21%	
S RC crncy	0,62%	
<b>Currency VA</b>	0,40%	

## **11. Financial market data applied for VA and MA calculation**

### **11.A. Market data for central government bonds**

275. The calculation of the LTAS is based on the basic risk-free interest rates term structures and the government yield curves described in section 3.

### **11.B. Financial market data for assets other than central government and central banks bonds**

#### **11.B.1. Market yields for corporate bonds**

276. The market yields for corporate bonds are those provided by the Markit – iBoxx indices listed in the tables below in this subsection. The yield is the '*annualized yield*' and the duration is the '*portfolio duration*' (rounded to the first decimal).

277. As mentioned above, the relevant yield curve is calculated by linear interpolation for those maturities provided by the source. For shorter and longer maturities the interest rate published for the nearest duration is applied.

278. Having in mind the availability of both the current value of market yield indices for exposures to corporate bonds, and of their historical series (necessary to calculate the long-term average spreads), the following decisions have been adopted for pragmatic reasons:

- a. CQS0 (AAA) corporate yield indices for the Euro and GBP have not been available during the last two years for a major part of the maturity buckets, and even for those maturity buckets where yields are available, the number of constituents of the index is very low. Furthermore, availability of buckets has continuously changed during the last years (i.e. not always the same buckets of duration have been available).

In order to solve the current lack of data and avoid the exposure of the calculation to likely business contingencies, the market yields of CQS0 exposures will be 85% of CQS1 yields for the Euro and for the GBP. The 0.85 reduction factor is based on the historical experience of those periods where both CQS0 and CQS1 yields have been simultaneously available.

- b. Regarding CQS1 non-financial bonds expressed in GBP, the available historical series of market yield indices for maturities from 4 to 9 years are incomplete and a reliable calculation of the long-term average spread (LTAS) is not possible.

Therefore for GBP Non financial bonds, credit quality step 1, the LTAS for maturities 4 to 8 years is obtained by linear interpolation of 3 and 9 years maturities LTAS. This rule does not apply to the current market yields, because for the time being it is possible to use the indices GBP CQS1 Non-financial.

- c. The currently available indices for CQS4 and CQS5 do not discriminate by duration. Therefore, the market yield of sub-investment grade assets CQS4 and CQS5 is used for all maturities(i.e. a flat curve is used).
- d. The market yield indices available for CQS6 are based on a limited number of constituents and the historical information available is not complete enough. For these exposures the market yield indices of CQS5 are applied.

279. EIOPA will monitor the effect of these criteria and the improvements of the available financial market data

#### 11.B.2. **Market data for the calculation of the PD and CoD**

280. The inputs necessary for the calculation of the probability of default and cost of downgrade are the benchmark curve used to calculate the spreads, the corporate bonds spreads to the benchmark curve, and the relevant transition matrices:

- a. The benchmark curve is the basic risk-free curve,
- b. The spreads are calculated as the difference between the market yields for corporate bonds described above, and the basic risk-free interest rate term structure. For the purposes of calibrating the multiplier described in subsection 9.C.2, this comparison is made using the 5 years term maturity (i.e. the spread curves are flat and parallel according to their level for 5 years maturity)
- c. Two transition matrices are used as inputs: financial and non-financial exposures. Both transition matrices have been obtained according to the following criteria:
  - i.) the transition probabilities refer to the 1 year average calculated along the last 30 years, until 1/1/2014;
  - ii.) having in mind the limited number of exposures per geographical area, credit quality step and economic sector, the geographical area considered refers to all countries;
  - iii.) the withdrawn exposures are excluded (i.e. not considered in the initial population of names);
  - iv.) the statistics refer to issuers (i.e. names);



- v.) having in mind the definition of the market source for ratings below CCC, those categories are included as defaults. Therefore matrices used as input have seven credit quality steps (i.e. eight rows and columns, including the situation of being defaulted, which is considered to be an absorbing state –no return to rated categories).

Markit – iBoxx indices	1-3yr	3-5yr	5-7yr	7-10yr	10+yr	10-15yr	15+yr
<b>EUR_Financial AAA</b>	85% of the EUR financial AA yields						
<b>EUR_Financial AA</b>	DE000A0JZBB2	DE000A0JZBD8	DE000A0JZBF3	DE000A0JZBH9	DE000A0JZA95		
<b>EUR_Financial A</b>	DE000A0JZA12	DE000A0JZA38	DE000A0JZA53	DE000A0JZA79	DE000A0JZAZ3		
<b>EUR_Financial BBB</b>	DE000A0JZBX6	DE000A0JZBZ1	DE000A0JZB11	DE000A0JZB37	DE000A0JZBV0		
<b>EUR_Financial BB</b>	Iboxx EUR High Yield curve Financial ex crossover LC BB (GB00B1CQYN32)						
<b>EUR_Financial B</b>	Iboxx EUR High Yield curve Financial ex crossover LC B (GB00B1CQYW23)						
<b>EUR_Financial CCC</b>	Iboxx EUR High Yield curve Financial ex crossover LC B (GB00B1CQYW23)						
<b>EUR_Non Financial AAA</b>	85% of the EUR Non financial AA yields						
<b>EUR_Non Financial AA</b>	DE000A0JZCH7	DE000A0JZCK1	DE000A0JZCM7	DE000A0JZCP0	DE000A0JZCF1		
<b>EUR_Non Financial A</b>	DE000A0JZB78	DE000A0JZB94	DE000A0JZCB0	DE000A0JZCD6	DE000A0JZB52		
<b>EUR_Non Financial BBB</b>	DE000A0JZC36	DE000A0JZC51	DE000A0JZC77	DE000A0JZC93	DE000A0JZC10		
<b>EUR_Non Financial BB</b>	Iboxx EUR High Yield curve Non-financial ex crossover LC BB (GB00B1CR1Z75)						
<b>EUR_Non Financial B</b>	Iboxx EUR High Yield curve Non-financial ex crossover LC B (GB00B1CR2653)						
<b>EUR_Non Financial CCC</b>	Iboxx EUR High Yield curve Non-financial ex crossover LC B (GB00B1CR2653)						
<b>GBP_Financial AAA</b>	85% of the GBP financial AA yields						
<b>GBP_Financial AA</b>	DE000A0JY7T1	DE000A0JY7X3	DE000A0JY7Z8	DE000A0JY712		DE000A0JY7R5	DE000A0JY7V7
<b>GBP_Financial A</b>	DE000A0JY7B9	DE000A0JY7F0	DE000A0JY7H6	DE000A0JY7K0		DE000A0JY696	DE000A0JY7D5
<b>GBP_Financial BBB</b>	DE000A0JY8R3	DE000A0JY8V5	DE000A0JY8X1	DE000A0JY8Z6		DE000A0JY8P7	DE000A0JY8T9
<b>GBP_Non Financial AAA</b>	85% of the GBP Non financial AA yields						
<b>GBP_Non Financial AA</b>	DE000A0JY9P5	DE000A0JY9T7	DE000A0JY9V3	DE000A0JY9X9		DE000A0JY9M2	DE000A0JY9R1
<b>GBP_Non Financial A</b>	DE000A0JY878	DE000A0JY9B5	DE000A0JY9D1	DE000A0JY9F6		DE000A0JY852	DE000A0JY894
<b>GBP_Non Financial BBB</b>	DE000A0JZAM1	DE000A0JZAR0	DE000A0JZAT6	DE000A0JZAV2		DE000A0JZAK5	DE000A0JZAP4

<b>Markit – iBoxx indices</b>	<b>1-3yr</b>	<b>3-5yr</b>	<b>5-7yr</b>	<b>7-10yr</b>	<b>10+yr</b>	<b>10-15yr</b>	<b>15+yr</b>
<b>USD_Financial AAA</b>	GB00B05DNG01	GB00B05DNH18	GB00B05DNJ32	GB00B05DNK47		GB00B05DNL53	GB00B05DNN77
<b>USD_Financial AA</b>	GB00B05DN483	GB00B05DN590	GB00B05DN608	GB00B05DN715		GB00B05DN822	GB00B05DNB55
<b>USD_Financial A</b>	GB00B05DMS57	GB00B05DMT64	GB00B05DMV86	GB00B05DMW93		GB00B05DMX01	GB00B05DN046
<b>USD_Financial BBB</b>	GB00B05DNS23	GB00B05DNT30	GB00B05DNV51	GB00B05DNW68		GB00B05DNX75	GB00B05DNZ99
<b>USD_Non Financial AAA</b>	GB00B05DQR21	GB00B05DQS38	GB00B05DQT45	GB00B05DQV66		GB00B05DQW73	GB00B05DQY97
<b>USD_Non Financial AA</b>	GB00B05DQD84	GB00B05DQF09	GB00B05DQG16	GB00B05DQH23		GB00B05DQJ47	GB00B05DQL68
<b>USD_Non Financial A</b>	GB00B05DQ270	GB00B05DQ387	GB00B05DQ494	GB00B05DQ502		GB00B05DQ619	GB00B05DQ833
<b>USD_Non Financial BBB</b>	GB00B05DR245	GB00B05DR351	GB00B05DR468	GB00B05DR575		GB00B05DR682	GB00B05DR807

For sub-investment grades bonds expressed in GBP and USD, the Euro rates will apply with the factor described in section 9.C.5

## **12. Calculation of the relevant risk-free interest rates term structures at a glance.**

281. The complete process of calculation may be summarized as follows:

### Basic risk-free interest rates term structure

Step A.- Use as input of the market interest rates for the relevant financial instrument according to section 3.C, table 1.

Step B.- According to the tables in section 4, removal of the rates either not meeting the DLT requirements (tables 3-4-5) or longer than the LLP (table 2).

Step C.- Calculation of the credit risk adjustment as described in section 5.

Step D.- Reduction of all the market rates remaining after step B by the amount of the credit risk adjustment (and the currency adjustment in the case of the Bulgarian and Danish currencies).

Step E.- Construction of the matrix of cash flows corresponding to the credit risk adjusted rates after step C.

One of the dimensions of this matrix reflects the maturities corresponding to DLT rates (e.g. from 1 to 20 years in the case of the Euro), while the other dimensions reflects the future terms with payments of the financial instrument applied in step 1, according to the frequency of the financial instrument (e.g. annualized rates in the case of the Euro curve). For simplicity, market conventions are not used, since its effect is negligible.

Step F.- Selection of the rest of inputs of the method of extrapolation in accordance with sections 4 and 6: LLP (table 2 and subsection 6.B), ultimate forward rate (subsection 6.D), convergence period, tolerance (1 basis point) and lower bound of alpha parameter (0.05) (subsection 6.E).

Step G.- Application of the method of extrapolation (annex to subsection 5.B).

### Risk-free interest rates term structure with the volatility adjustment

Step H.- Calculation of the volatility adjustment. This subprocess has been described in section 10 above. For each relevant currency and each relevant country, the volatility adjustment is a fixed number, expressed in basis points and rounded to the nearest integer basis point, and applied to all maturities till the last liquid point.

Step I.- Construction of the matrix of cash flows corresponding to the zero-coupon annualized rates resulting from step G. All integer maturities until the last liquid point, included, will be used to build this

matrix. Furthermore, for each maturity a single payment will be considered. Therefore the matrix of this step will usually have different dimensions than the one built in step E.

Step J.- Addition of the annualized volatility adjustment to the matrix of cash flows obtained in step H.

Step K.- Application of the method of extrapolation with the same inputs used in step F and according to the method mentioned in step G.

282. The volatility adjustment is not added directly to the par swap rates adjusted for credit risk but is added to the zero-coupon spot rates of the basic risk-free interest rate term structure obtained after using the Smith-Wilson method (as described in an earlier part of this Technical Documentation).
283. In accordance with Article 46 of the Delegated Regulation, the volatility adjustment is added to the aforementioned zero-coupon spot rates only in the liquid part of the curve.
284. The resulting rates are the relevant risk-free interest rates including the volatility adjustment to which the extrapolation is applied, using again the Smith-Wilson method.
285. Because the volatility adjustment is applied to the liquid zero coupon rates of the basic risk-free interest rate term structure, the relevant risk-free interest rate term structure including the VA is a parallel shift of the basic risk-free interest rate term structure until the LLP. There is no parallel shift after the LLP since both the basic and relevant risk-free curves ultimately converge to the same UFR.

# Annexes

## 13. Annexes of Section 3.

### 13.A. Disclaimers

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### **13.B. Annex to Section 3: Selection of reference instruments and DLT assessment**

286. Solvency II sets out market consistency as a core principle for the assessment of the financial and solvency position of insurance and reinsurance undertakings. The principle of market consistency applies for both assets and liabilities<sup>23</sup>. In particular, the calculation of technical provisions should use the relevant risk-free interest rate term structure, which should be based on upon up-to-date and credible information<sup>24</sup>.
287. These principles underpin the assessment of the depth, liquidity and transparency (DLT) of observable market interest rates. As well as providing assurance that the relevant DLT requirements are met, the DLT assessment should foster the optimal use of the information provided by financial markets<sup>25</sup>.
288. In developing the methodology applied for the DLT assessment, EIOPA has analysed the generally applied practices and the academic literature on the issue. This analysis has looked in particular at the process of liquidity assessment, but has also considered the available measures of depth and transparency.
289. As part of the preparation and follow-up of the Long-term Guarantees Assessment, during 2013 EIOPA developed a conceptual framework for DLT assessment based on the aforementioned analysis. This conceptual framework was put into practice on a tentative basis for the EIOPA Stress Test 2014.
290. EIOPA's work and lessons learnt during 2013 are in alignment with the EBA's report on appropriate uniform definitions of extremely high quality liquid assets (extremely HQLA) and high quality liquid assets (HQLA) and on operational requirements for liquid assets under Article 509(3) and (5) CRR (20 December 2013)<sup>26</sup>.
291. While recognizing the differences between the banking and insurance sectors, EIOPA recognises the existence of commonalities between the DLT assessment for risk-free interest rate term structures and the work carried out by EBA on extremely HQLA and HQLA.

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<sup>23</sup> Recital 53, Article 75 and 76 of Directive 2009/138/EC

<sup>24</sup> Recital 58 and Article 77 of Directive 2009/138/EC

<sup>25</sup> Recital 45 Directive 2009/138/EC

<sup>26</sup> EBA report on extremely HQLA and HQLA



292. Although there is a set of generally applied metrics for the purpose of making a DLT assessment, practitioners, academics and regulators/supervisors are currently constrained by the following limitations:
293. While there is a general approach to assessing liquidity and depth, the precise definitions of these terms depend on the context. For example, the definition of 'liquidity' for the purpose of the Liquidity Coverage Ratio (LCR) in the banking sector is quite similar to its definition in the case of the DLT assessment in the insurance sector. With that said, the purpose of the DLT assessment is focused on ensuring the reliability of market interest rates rather than the need to convert assets into cash.
294. There are several factors influencing the liquidity (and depth) of financial markets. Further, the influence of these factors varies across markets (e.g. according to their practices, conventions and operational rules), and also varies overtime within the same market (e.g. according to changes in the environment). Finding a generalized way to measure the level of these factors is the subject of continuing research.
295. It is generally accepted that no single metric can be conclusive in assessing the DLT nature of a financial instrument. For example, high trading volumes and turnovers indicate that assets are liquid, while the converse does not necessarily hold true (some assets may be in high demand without being traded often, and hence could be easily liquidated if needed<sup>27</sup>).
296. There are severe limitations for the calculation of some metrics, in terms of the availability and reliability of the inputs necessary for the calculation and the completeness or homogeneity of the data series. In particular, for the swap market, the lack of information on real trading volumes means that it is not possible to use some of the main indicators generally used when making DLT assessments of other types of instrument. This limitation has particular importance because Solvency II's legal provisions prescribe swaps as the first choice of instrument for deriving the relevant risk-free interest rate term structure.
297. Finally, practitioners, academics and supervisors acknowledge the relevance of supplementing quantitative metrics with a qualitative / expert judgement. EIOPA supports the appropriate consideration of qualitative information, and this view is also reflected in the

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<sup>27</sup> EBA report on extremely HQLA and HQLA (pg 16)

aforementioned EBA report<sup>28</sup>. In particular, EIOPA is of the view that the assessment of the depth of a financial market should take into account the existence of appropriate supervision; such supervision can be an effective mechanism to ensure that large transactions will only affect prices according to the natural trends of the market, and not because of any spurious influence. Another relevant qualitative consideration for the assessment of market depth is the way in which market prices are collected; market data providers have developed effective methods and controls that can help to give reassurance that the influence of large transactions or unusual trades on prices is likely to be immaterial.

298. The following annexes describe EIOPA's approach to DLT assessment, for the two separate cases below:
- a. EEA currencies, for which it is feasible to obtain ad-hoc information on pricing and trading (except for traded volumes for swaps, as mentioned above).
  - b. Non-EEA currencies, for which EIOPA has adapted its methodology to account for data limitations. In particular this approach includes those metrics used by the EBA that do not rely on either traded volumes or on any other information that is not generally available.
299. In both cases, EIOPA's methodology aims to provide a stable DLT assessment; this is considered a necessary condition to allow insurance and reinsurance undertakings to implement the relevant calculation processes. Therefore, as a general rule, hard thresholds and the automatic use of benchmarks have not been considered appropriate. For example, comparing the bid-ask spreads of one currency against another does not necessarily provide conclusive evidence for a DLT assessment, not only because of the specifics of each financial market (level of interest rates, trends, etc.), but also because experience shows that the relative positions of two currencies may change over time.

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<sup>28</sup> EBA report on extremely HQLA and HQLA (pg 26)

### **13.C. Annex of Subsection 3.D. DLT assessment of EEA currencies**

300. The DLT assessment for EEA currencies is based on the conceptual framework that EIOPA developed in 2013.
301. As mentioned in the general annex to Section 3 above, each of the Depth, Liquidity and Transparency criteria lacks a globally accepted clear definition that is of practical use. Even in academic literature, a wide range of measures for depth and liquidity exist; however, none of those is considered authoritative and applicable in all markets.
302. Therefore, the list of criteria mentioned below should be considered as non-exhaustive. EIOPA has focused on criteria that may be helpful in assessing the credibility of market data for interest rate swaps and government bonds. Additional criteria consider the general bond market. The criteria are as follows:
- a. Bid-ask spread (BAS): the price difference between the highest price a buyer would pay and the lowest price for which a seller would settle;
  - b. Trade frequency (TF): number of trades that take place within a defined period of time;
  - c. Trade volume (TV);
  - d. Trader quotes/dealer surveys (TQDS) (incl. dispersion of answers);
  - e. Quote counts (1) (QC1): number of dealer quotes within a few day window;
  - f. Quote counts (2) (QC2): number of dealers quoting;
  - g. Number of pricing sources (NPS);
  - h. Assessment of large trades and movement of prices (depth) (XLT);
  - i. [Only applicable to the Euro] Residual volume approach (for bonds only) (RVA).

### 13.D. Annex of Subsection 3.D: DLT assessment of non-EEA currencies

303. The DLT assessment of non-EEA currencies is based, in addition to qualitative analysis, on the joint consideration of three main methodologies:
- volatility analysis;
  - analysis of bid-ask spreads (both direct observations and also using the Roll measure, as described below);
  - quantitative analysis.
304. DLT assessment methodology presented in this annex is going to be used for non-EEA currencies. DLT assessment for EEA currencies is presented only for illustration purposes, since DLT assessment for EEA currencies will be conducted according to methodology described in subsection 4.B.

#### 13.D.1. Volatility analysis

305. For the volatility analysis, the behaviour of the available interest rates for each maturity and non-EEA currency over the past 105 business days is analysed (this is approximately a chronological period of five months previous to the reference date of the term structure).
306. The analysis is conducted for rates directly observed in markets (e.g. par swap rates where swaps are the financial instrument used as reference), for zero-coupon spot rates, and finally for the 1-year forward rate term structure.
307. For each of the three sets of rates above, and for each currency and maturity, the analysis considers both the values of the rate and the behaviour of the volatility calculated considering the last 21 days<sup>29</sup>(approximately one chronological month). Therefore, 84 values

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<sup>29</sup> The following formulation is used:

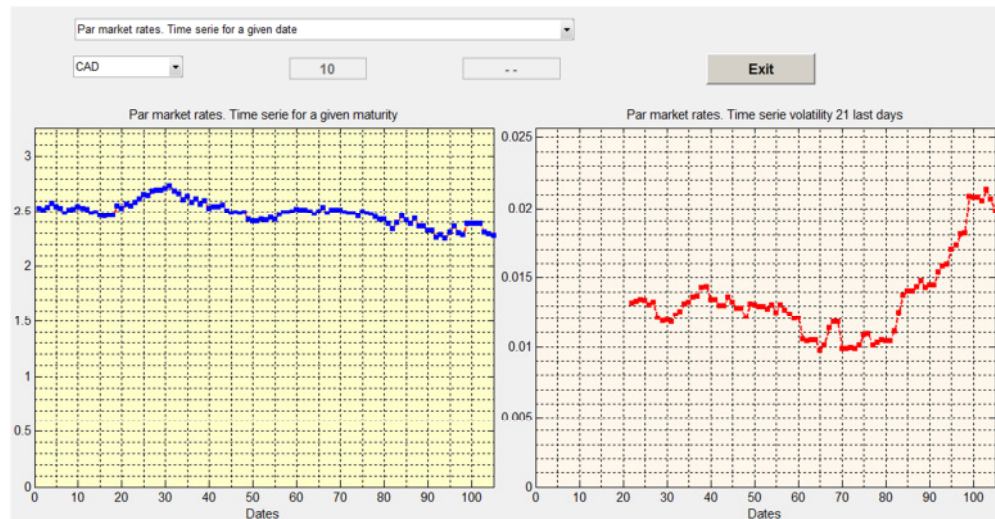
Volatility = standard deviation of natural logarithms of variations =

$$= \sqrt{\sum \frac{(\ln(c_i) - \overline{\ln c})^2}{n-1}} \quad \text{where } \ln c_k = \ln \frac{\text{rate}_k}{\text{rate}_{k-1}} \text{ and } \overline{\ln c} \text{ identifies the simple}$$

average of last 21 daily logarithmic changes,

of the volatility for each rate are calculated, with rolling windows referring to the last 105 trading days (i.e. for the oldest 21 dates in the series, no volatility is calculated, as these dates do not have the 21-day period of reference necessary for the calculation).

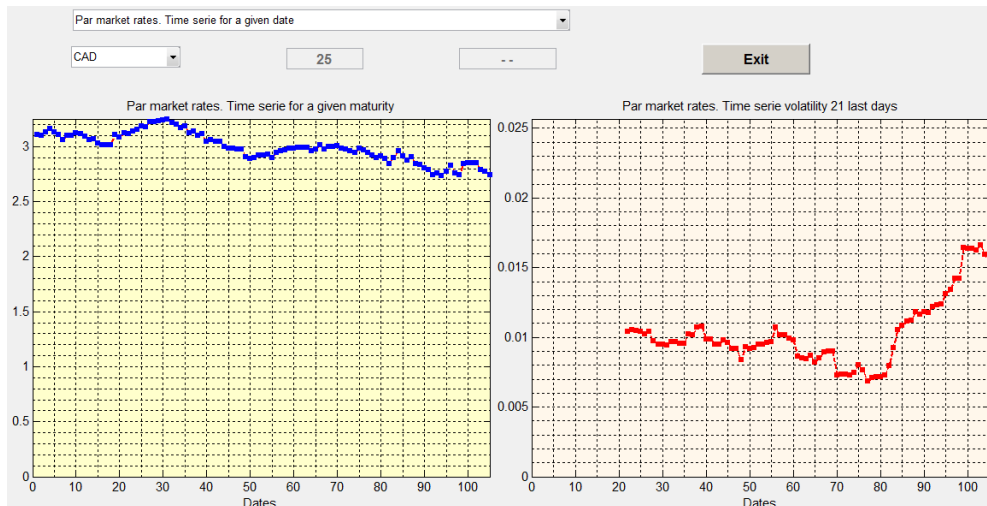
308. The analysis described in the paragraphs above is used to conduct three tests and to produce the set of statistics described below.
309. The first test focuses on how the rate for a given maturity behaves during the 105 day window (both the level of the rate itself and its 21-day volatility).
310. As an example, the charts below show the behaviour of the 10-year (first two charts) and 25-year rates and volatilities (second two charts) for the Canadian dollar, as of 31-12-2014, using the par swap market rates.



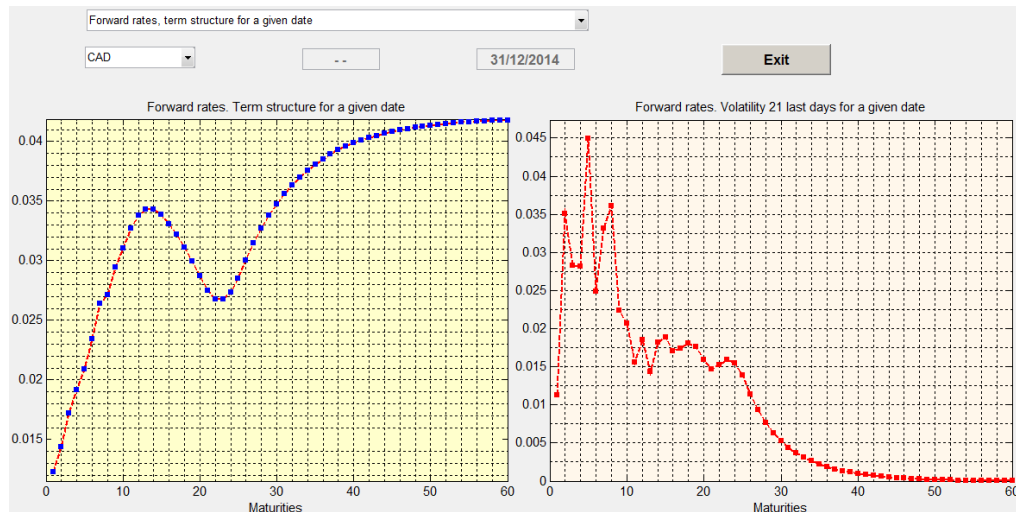
311. There are several ways of inferring an empirical view on the behaviour of the interest rates. For example, by considering the values of the rates (y-axis in the left chart) and the level of the volatility (y-axis on the right hand side), by considering the lack of/presence of repeated sudden changes in the level of the volatility, or by examining the range of variation in both charts. From these perspectives the rates for both maturities show a similar pattern, and do not convey abnormal features.

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Note that no  $\sqrt{t}$  adjustment is applied in order to achieve annual volatilities. This has no impact of the conclusions to the extent the DLT analysis aims at comparing volatilities, not at assessing its values on annual basis.

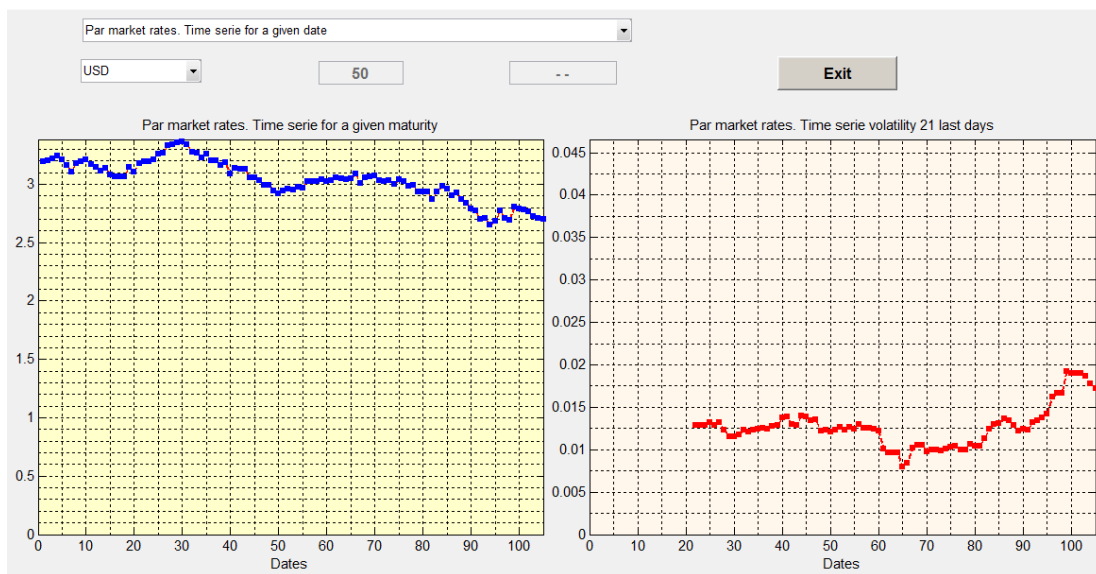
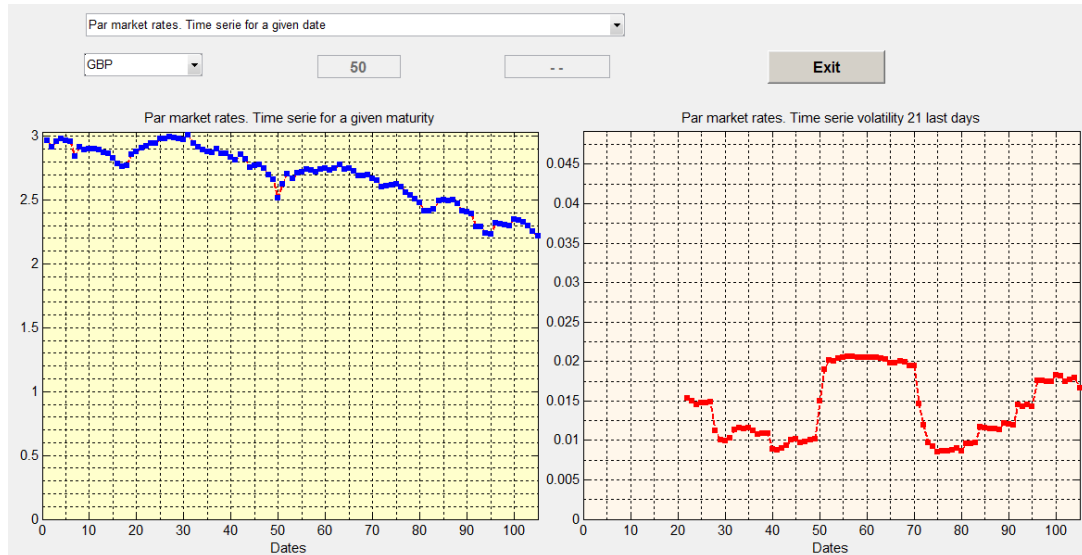


312. The second test aims to detect whether the rate for a given maturity produces humps or hollows in the term structure curve (i.e. by comparing with the behaviour of neighbouring maturities).
313. Again using the example of the Canadian curve as at 31-12-2014, it can be seen that the curve does not present abnormal features and the 21-days volatility of all observable maturities is in a reasonable range (note the LLP for the Canadian currency is 25 years, therefore the part of the curve for maturities longer than 25 years does not represent market data, but the Smith-Wilson extrapolation).

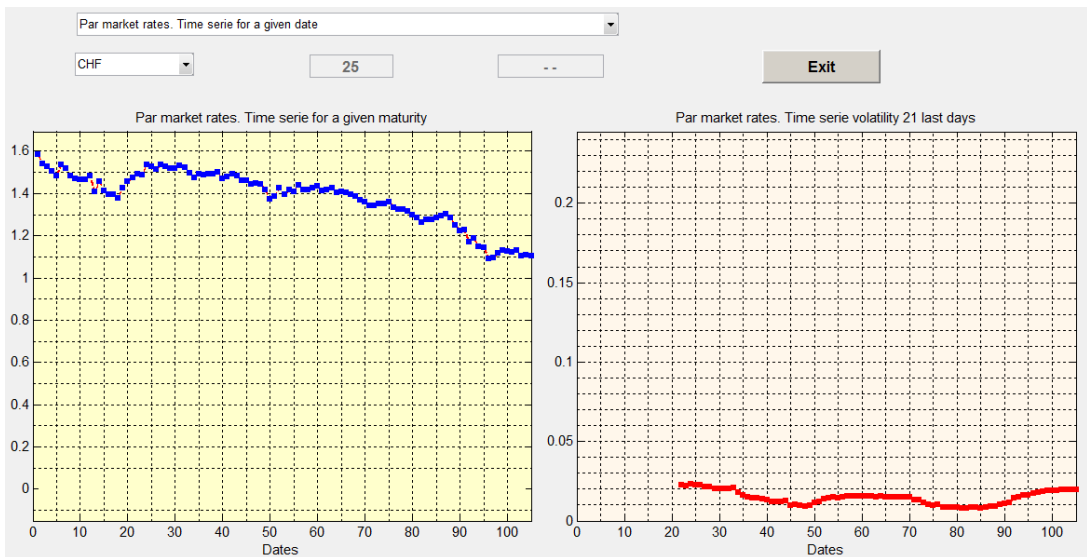
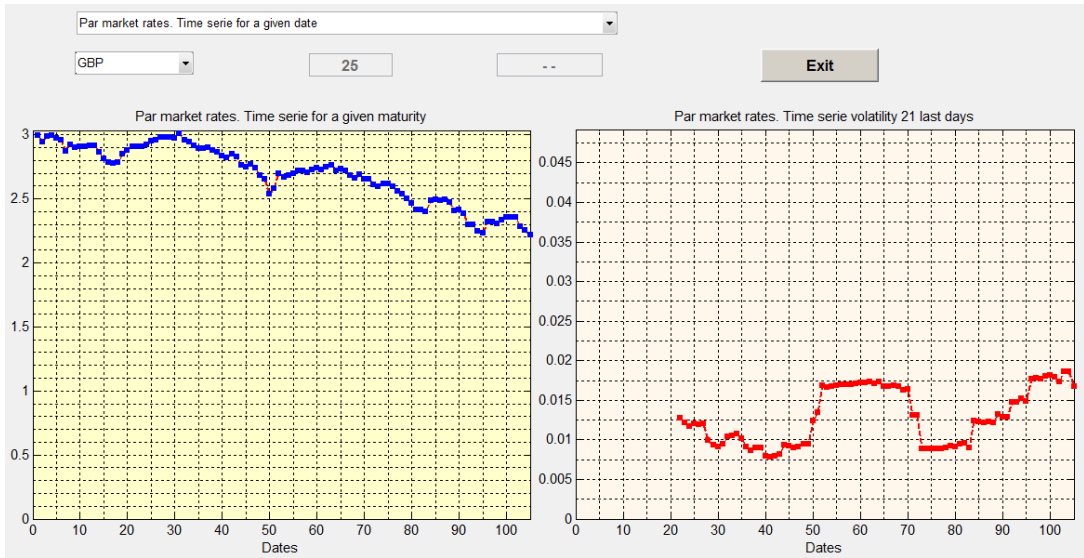


314. For the third analysis, a comparison across currencies has been developed. The comparison is used in situations where there is an adequate relationship between the non-EEA currency now being analysed and an EEA currency whose DLT nature has been tested as described in section 3C . This third test aims to verify whether the behaviour of the non-EEA rate is sufficiently similar to its 'peer' EEA rate.

315. For example, the charts below compare the behaviour of 50-year maturities for GBP and USD as at 31-12-2014 using par swap rates (note that the similarity of behaviours between these currencies is also observed when using zero-coupon rates and forward rates).



316. The charts below compare the behaviour of 25-year maturities for GBP and CHF as at 31-12-2014 using 1-year forward rates (note that the similarity of behaviours is also observed when using par swap rates and zero-coupon rates).





### 13.D.2. The analysis of bid-ask spreads: Direct observation

317. For all currencies where a 'likely' longest DLT maturity has been established, a direct investigation of the specific bid-ask spreads at these maturities is also carried out. The following metrics are obtained for the month prior to the reference date and also for the last quarter:

- Median of bid-ask spreads during the last month;
- 80<sup>th</sup> Percentile of bid-ask spreads during the last month;
- Maximum of bid-ask spreads during the last month;
- Simple Average of bid-ask spreads during the last month;
- Last spread (at the date of reference of the curve);
- Number of days with zero spreads.

318. The tables below summarizes some findings for long-term maturities of swaps as of 31-12-2014 (currencies identified according to ISO3166 in all tables):

#### Analysis of bid-ask spread for 15-year interest rates swaps IBOR

	Last 64 days with trading						Last 21 days with trading					
	Zero observations	Median non-zero spreads	Percentile 80 non-zero spreads	Maximum spread	Average non-zero spreads	Last non-zero spread	Zero observations	Median non-zero spreads	Percentile 80 non-zero spreads	Maximum spread	Average non-zero spreads	Last non-zero spread
EUR	0	2.25	4.00	4.00	2.34	2.40	0	3.00	4.00	4.00	2.58	2.40
BGN	48	260.00	260.00	260.00	260.00	260.00	5	260.00	260.00	260.00	260.00	260.00
CZK	0	4.00	4.00	7.00	4.22	4.00	0	4.00	4.00	4.00	3.86	4.00
DKK	0	3.00	3.00	7.00	3.06	7.00	0	3.00	4.73	7.00	3.70	7.00
HUF	3	6.00	7.45	10.00	5.93	10.00	0	6.00	6.00	10.00	5.21	10.00
LIC	0	4.00	4.00	10.00	3.61	4.00	0	4.00	4.00	10.00	3.62	4.00
NOK	0	10.00	10.00	10.00	7.54	4.00	0	5.00	10.00	10.00	6.44	4.00
PLN	0	3.00	4.00	6.00	3.37	3.00	0	3.00	3.00	3.00	3.00	3.00
RON	0	140.00	140.00	140.00	140.00	140.00	0	140.00	140.00	140.00	140.00	140.00
RUB	0	14.00	14.00	14.00	14.00	14.00	0	14.00	14.00	14.00	14.00	14.00
SEK	0	3.00	3.10	6.00	3.17	3.00	0	3.00	3.00	3.10	2.88	3.00
CHF	0	4.00	4.00	10.00	3.61	4.00	0	4.00	4.00	10.00	3.62	4.00
GBP	0	1.00	1.00	1.90	1.00	1.80	0	1.00	1.56	1.90	1.13	1.80
AUD	0	3.00	4.00	8.50	3.45	4.00	0	3.00	4.00	8.50	3.60	4.00
CAD	0	3.17	4.00	6.10	3.09	4.00	0	3.42	4.00	5.90	3.06	4.00
CLP	2	4.00	5.00	5.00	4.21	4.00	2	4.00	4.70	5.00	4.21	4.00
CNY	3	40.00	40.00	59.00	40.51	40.00	0	39.00	40.00	59.00	40.33	40.00
HKD	2	7.00	7.10	10.00	7.25	10.00	2	7.00	7.10	10.00	7.48	10.00
JPY	0	2.00	2.00	8.00	2.20	2.00	0	2.00	2.00	8.00	2.24	2.00
MYR	0	10.00	10.00	10.00	9.94	10.00	0	10.00	10.00	10.00	10.00	10.00
MXN	0	4.00	4.00	6.00	4.00	4.00	0	4.00	4.60	6.00	4.27	4.00
NZD	3	1.00	1.74	8.00	2.00	0.75	1	0.75	1.00	8.00	1.39	0.75
SGD	0	7.00	7.00	7.10	5.88	7.00	0	7.00	7.00	7.00	6.12	7.00
ZAR	0	8.00	10.00	10.00	8.06	8.00	0	8.00	10.00	10.00	8.25	8.00
KRW	0	3.00	3.50	3.50	3.20	3.00	0	3.00	3.50	3.50	3.17	3.00
THB	0	9.00	10.00	12.00	8.84	9.00	0	9.00	10.00	10.00	8.71	9.00
TRY	0	40.00	40.00	42.00	40.08	40.00	0	40.00	40.00	41.00	40.05	40.00
USD	2	0.40	0.51	0.80	0.39	0.50	-	0.45	0.56	0.80	0.43	0.50

#### Analysis of bid-ask spread for 20-year interest rates swaps IBOR

	Last 64 days with trading						Last 21 days with trading					
	Zero observations	Median non-zero spreads	Percentile 80 non-zero spreads	Maximum spread	Average non-zero spreads	Last non-zero spread	Zero observations	Median non-zero spreads	Percentile 80 non-zero spreads	Maximum spread	Average non-zero spreads	Last non-zero spread
EUR	0	2.18	4.00	4.00	2.28	2.40	0	2.66	4.00	4.00	2.52	2.40
BGN	48	260.00	260.00	260.00	260.00	260.00	5	260.00	260.00	260.00	260.00	260.00
CZK	0	4.00	4.00	7.00	4.20	4.00	0	4.00	4.00	4.00	3.86	4.00
DKK	0	3.00	3.00	7.00	3.12	7.00	0	3.00	4.80	7.00	3.63	7.00
HUF	4	6.00	10.00	10.00	6.67	6.00	1	6.00	6.00	6.00	5.73	6.00
LIC	0	4.00	4.00	10.00	3.66	5.00	0	3.00	4.30	10.00	3.33	5.00
NOK	0	15.50	15.50	15.50	14.66	3.70	0	15.50	15.50	15.50	12.94	3.70
PLN	1	3.00	3.97	6.00	3.25	3.00	0	3.00	3.00	3.00	2.89	3.00
RON	0	140.00	140.00	140.00	140.00	140.00	0	140.00	140.00	140.00	140.00	140.00
RUB	2	10.00	60.00	61.00	24.53	60.00	2	60.00	60.70	61.00	55.42	60.00
SEK	0	3.00	3.10	8.00	3.57	3.00	0	3.00	3.10	6.00	3.15	3.00
CHF	0	4.00	4.00	10.00	3.66	5.00	0	3.00	4.30	10.00	3.33	5.00
GBP	0	1.18	1.33	13.10	1.69	1.16	0	1.18	1.45	13.10	2.68	1.16
AUD	0	3.68	4.00	4.00	3.67	4.00	0	3.62	4.00	4.00	3.65	4.00
CAD	0	3.38	4.00	6.10	3.10	4.00	0	3.93	4.05	6.10	3.28	4.00
CLP	2	4.00	5.00	5.00	4.21	4.00	2	4.00	5.00	5.00	4.26	4.00
JPY	0	2.00	2.00	8.00	2.20	2.00	0	2.00	2.00	8.00	2.24	2.00
MYR	0	10.00	10.00	10.00	9.97	10.00	0	10.00	10.00	10.00	10.00	10.00
MXN	20	3.00	3.00	3.00	3.00	3.00	14	3.00	3.00	3.00	3.00	3.00
NZD	0	1.00	5.73	8.00	2.36	8.00	0	1.00	3.10	8.00	2.25	8.00
SGD	0	7.00	7.00	7.70	6.10	7.00	0	7.00	7.00	7.70	6.39	7.00
ZAR	0	8.00	8.00	11.00	7.24	8.00	0	8.00	8.00	11.00	8.00	8.00
KRW	0	3.25	3.50	3.50	3.24	3.50	0	3.50	3.50	3.50	3.31	3.50
THB	0	15.00	15.00	31.50	13.01	6.00	0	15.00	15.00	15.00	12.00	6.00
TRY	0	20.00	20.00	21.00	20.02	20.00	0	20.00	20.00	21.00	20.05	20.00
USD	0	0.40	0.55	0.80	0.41	0.50	-	0.48	0.57	0.80	0.44	0.50

### Analysis of bid-ask spread for 25-year interest rates swaps IBOR

	Last 64 days with trading						Last 21 days with trading					
	Zero observations	Median non-zero spreads	Percentile 80 non-zero spreads	Maximum spread	Average non-zero spreads	Last non-zero spread	Zero observations	Median non-zero spreads	Percentile 80 non-zero spreads	Maximum spread	Average non-zero spreads	Last non-zero spread
EUR	0	2.46	4.00	4.00	2.43	2.40	0	3.00	4.00	4.00	2.56	2.40
CZK	0	4.00	4.00	7.00	4.45	4.00	0	4.00	4.00	4.00	4.00	4.00
DKK	0	3.00	3.00	7.00	3.25	7.00	0	3.00	4.20	7.00	3.76	7.00
LIC	0	6.00	6.00	10.00	4.48	5.00	0	3.00	5.30	10.00	3.41	5.00
NOK	0	17.50	17.50	21.30	16.94	3.70	0	17.50	17.50	17.50	15.60	3.70
SEK	1	3.00	5.00	8.00	3.77	3.00	1	3.00	5.00	8.00	3.56	3.00
CHF	0	6.00	6.00	10.00	4.48	5.00	0	3.00	5.30	10.00	3.41	5.00
GBP	0	1.00	1.00	1.70	1.00	1.00	0	1.00	1.00	1.70	1.02	1.00
AUD	0	3.56	4.00	4.00	3.66	4.00	0	3.62	4.00	4.00	3.65	4.00
CAD	0	3.00	4.00	6.10	3.04	4.00	0	3.45	4.00	6.00	3.25	4.00
JPY	0	2.00	2.00	8.00	2.39	8.00	0	2.00	2.00	8.00	2.47	8.00
MYR	0	10.00	10.00	10.00	9.97	10.00	0	10.00	10.00	10.00	10.00	10.00
NZD	7	1.00	1.00	1.00	1.00	1.00	0	1.00	1.00	1.00	1.00	1.00
ZAR	0	8.00	10.00	10.00	7.95	8.00	0	8.00	10.00	10.00	8.40	8.00
KRW	0	3.50	3.50	3.50	3.26	3.50	0	3.50	3.50	3.50	3.33	3.50
USD	0	0.40	0.60	0.85	0.45	0.60	0	0.50	0.60	0.69	0.48	0.60

## Analysis of bid-ask spread for 30-year interest rates swaps IBOR

	<i>Last 64 days with trading</i>						<i>Last 21 days with trading</i>					
	<i>Zero observations</i>	<i>Median non-zero spreads</i>	<i>Percentile 80 non-zero spreads</i>	<i>Maximum spread</i>	<i>Average non-zero spreads</i>	<i>Last non-zero spread</i>	<i>Zero observations</i>	<i>Median non-zero spreads</i>	<i>Percentile 80 non-zero spreads</i>	<i>Maximum spread</i>	<i>Average non-zero spreads</i>	<i>Last non-zero spread</i>
EUR	0	3.00	3.00	3.00	2.28	1.00	0	3.00	3.00	3.00	2.39	1.00
CZK	0	4.00	4.00	7.00	3.95	4.00	0	4.00	4.00	4.00	3.81	4.00
DKK	0	3.00	3.00	7.00	2.94	7.00	0	3.00	4.80	7.00	3.43	7.00
LIC	0	6.00	6.00	10.00	4.91	6.00	0	6.00	6.00	10.00	4.66	6.00
NOK	0	17.50	17.50	17.50	16.55	3.70	0	17.50	17.50	17.50	15.77	3.70
PLN	0	4.00	4.00	6.00	4.08	4.00	0	4.00	4.00	4.00	4.00	4.00
SEK	0	5.37	6.00	10.00	5.05	3.00	0	3.00	6.00	10.00	4.57	3.00
CHF	0	6.00	6.00	10.00	4.91	6.00	0	6.00	6.00	10.00	4.66	6.00
GBP	0	0.95	1.00	2.00	0.98	1.80	0	1.00	1.83	2.00	1.17	1.80
AUD	0	3.75	4.00	5.00	3.64	4.00	0	3.75	4.00	4.00	3.68	4.00
CAD	0	4.00	4.00	6.10	3.92	4.00	0	4.00	4.00	6.10	4.00	4.00
JPY	0	2.00	2.00	8.00	2.40	8.00	0	2.00	2.00	8.00	2.52	8.00
SGD	0	5.00	6.00	8.00	5.24	6.00	0	5.00	6.00	8.00	5.33	6.00
ZAR	0	8.00	8.00	10.00	7.53	8.00	0	8.00	8.00	10.00	7.86	8.00
KRW	0	3.50	3.50	3.50	3.26	3.50	0	3.50	3.50	3.50	3.33	3.50
USD	1	0.44	0.70	1.45	0.47	0.72	0	0.48	0.71	0.92	0.49	0.72

## Analysis of bid-ask spread for 50-year interest rates swaps IBOR

	<i>Last 64 days with trading</i>						<i>Last 21 days with trading</i>					
	<i>Zero observations</i>	<i>Median non-zero spreads</i>	<i>Percentile 80 non-zero spreads</i>	<i>Maximum spread</i>	<i>Average non-zero spreads</i>	<i>Last non-zero spread</i>	<i>Zero observations</i>	<i>Median non-zero spreads</i>	<i>Percentile 80 non-zero spreads</i>	<i>Maximum spread</i>	<i>Average non-zero spreads</i>	<i>Last non-zero spread</i>
EUR	0	3.00	3.00	3.00	2.39	2.00	0	3.00	3.00	3.00	2.49	2.00
LIC	0	10.00	10.00	10.00	10.00	10.00	0	10.00	10.00	10.00	10.00	10.00
CHF	0	10.00	10.00	10.00	10.00	10.00	0	10.00	10.00	10.00	10.00	10.00
GBP	0	1.00	1.00	2.00	1.07	1.80	0	1.00	1.52	2.00	1.21	1.80
USD	0	1.20	2.00	2.10	1.36	2.00	0	1.20	2.00	2.00	1.45	2.00

### 13.D.3. **The analysis of bid-ask spreads: Roll measure (all currencies)**

319. For this analysis, EIOPA has followed the approach used in the EBA report on extremely HQLA and HQLA. According to the EBA report:

Roll (1984)<sup>30</sup> shows that under certain conditions, the percentage bid/ask spread equals two times the square root of minus the covariance between consecutive returns:

$$Roll_{i,t} = 2 * \sqrt{-cov(r_{i,t}, r_{i,t-1})} , \text{ where}$$

$t$  is the time period over which the measure is calculated. The higher value of Roll measure, the lower liquidity of the analysed interest rate,

$$r_x = \text{price}_k - \text{price}_{k-1}$$

320. EIOPA's analysis considers a daily Roll measure, using a 21 trading day rolling window in the computation of the covariance. In cases where a positive covariance is found, the Roll measure is set to zero.

321. The set of analytical tests described for the volatility analysis are also applied for the Roll measure, although in this case only the zero coupon rates are examined. This approach (examining only the zero coupon rates) does not have a material influence on the outcome of the assessment, because all the information is already captured in the chart analysis for both the volatility and the Roll measurement.

### 13.D.4. **Quantitative analysis**

322. As mentioned in Annex 3, EIOPA does not consider it appropriate to apply hard thresholds purely based on quantitative metrics, because it is necessary to make an appropriate allowance for the characteristics of each individual market and for prevailing financial conditions.

323. For the same reasons, metrics that can be calculated as at a specific date should be supplemented by examining the behaviour of these metrics during the rolling windows of the period of observation mentioned above (105 days).

324. Thus, additional relevant metrics are as follows:

- a. Number of days without any available data;

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<sup>30</sup>ROLL, R. (1984), A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. The Journal of Finance, 39: 1127-1139.

- b. Median of spot zero coupon rates during the 105 day period of observation. This provides a metric to measure the 'size effect', which is currently material both across currencies and across maturities within the same currency.
  - c. Trend of interest rates during the period (obtained as the first degree coefficient of a linear fitting with LSM). This metric is necessary for an appropriate assessment of other metrics, to the extent that the existence of a clear and strong trend in interest rates, influences other metrics (e.g. the Roll measure).
  - d. For the series of zero coupon rates, the interquartile range ( $Q_{75} - Q_{25}$ ) relative to the median.
  - e. For the series of zero coupon rates, the number of outliers, calculated as the number of interest rates falling outside of the interval (mean - 1.5 standard deviations; mean + 1.5 standard deviations). Note that these statistics are calculated using only the interest rates between the 12.5<sup>th</sup> and 87.5<sup>th</sup> percentiles (thus avoiding any influence on the mean or standard deviation of 'large' outliers).
  - f. Last 21-day volatility observed in the 105 day period.
  - g. For the series of first order differences of zero coupon rates, the interquartile range ( $Q_{75} - Q_{25}$ ) relative to the median.
  - h. For the series of first order differences of zero coupon rates, the number of outliers as described above.
  - i. Last observed Roll measure.
  - j. 90<sup>th</sup> Percentile for the series of Roll measurements.
  - k. 90<sup>th</sup> Percentile of logarithmic returns.
325. The table below provides an illustrative example of the outputs of these metrics, for those non-EEA currencies where it has been possible to obtain interest rates for 40-year maturities<sup>31</sup>.
326. As mentioned above, this quantitative analysis is supplemented with the other analysis mentioned in this annex.

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<sup>31</sup> Value 65.535 identifies "Not a number" result (e.g. when data lead to a division by zero).

	Zero observations	Median zero spot rate	Linear growth	Interquartile range / Median for rates	Num. Rates Outliers	Last 21-days volat	Interquartile range / Median for dijfs	Num. Dijfs Outliers	Last Roll measure	Perctile 90 Roll	Perctile 90 log returns
EUR	0	2.20	- 0.47	12.61	10.00	2.00	- 5.95	18.00	34.41	47.00	1.72
CHF	0	1.77	- 0.25	10.13	14.00	1.65	- 7.93	16.00	28.70	5.17	1.25
GBP	0	3.21	- 0.46	9.10	1.00	1.21	- 24.80	11.00	-	62.20	1.42
AUD	0	4.52	- 0.18	5.87	12.00	1.90	- 21.41	15.00	62.50	68.79	3.38
JPY	0	2.02	- 0.10	3.40	6.00	1.16	- 14.12	13.00	18.97	36.74	0.80
USD	0	3.49	- 0.28	5.25	16.00	1.43	- 19.69	15.00	33.26	41.08	2.29

## 14. Annex of Subsection 6.A.

### 14.A. Description of the Smith-Wilson method with intensities

#### 1. An Interest Trinity

We consider an annual interest *rate*  $r$  that defines an annual interest *factor*  $R=(1+r)$ . From this we define a continuous-time interest *intensity*  $\rho=\log(R)$ . Negative interest rates are allowed, but we should have  $r>-1$  or  $R>0$ . Only the interest intensity  $\rho$  is unrestricted and this makes it convenient for modelling purposes. We will use the concise term intensity instead of instantaneous rate or infinitesimal rate to avoid ambiguity with annualised interest rates.

#### 2. Another Trinity

With a constant  $\rho$  the *present value* of an amount of 1 maturing after  $v$  years would be just  $p(v)=\exp(-v\rho)$ . Now interest intensities are seldom flat, so it is of interest to analyse present value with changing interest intensity. The *yield intensity function* is what would be the average flat interest intensity:

$$(2.1) \quad p(v) = \exp(-v \cdot y(v)) \quad \Leftrightarrow \quad y(v) = \frac{-\log p(v)}{v}$$

The *forward intensity function* measures the change in the present value function:

$$(2.2) \quad f(v) = \frac{-d \log p(v)}{dv} = \frac{-p'(v)}{p(v)}$$

The yield function can also be written as an averaged integral of the forward function:

$$(2.3) \quad y(v) = \frac{1}{v} \int_0^v f(z) dz$$

For the forward and yield curve there holds that  $y(0)=f(0)$ , the zero spot intensity. Also in the limit we will have  $y(\infty)=f(\infty)$ , the ultimate forward intensity. Furthermore any turning point of the yield curve will be crossed by the forward curve. This similarity with average and marginal cost curves is mentioned by McCulloch (1971), page 24.<sup>32</sup>

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<sup>32</sup>McCulloch, J Huston, 1971. "Measuring the term structure of interest rates". *The Journal of Business*, University of Chicago Press vol.44 (1) 19-31 January.

A parallel shock in the forward intensity curve will translate as the same parallel shock in the yield intensity curve. This property does not transpose to annualised interest rates, however.

### 3. A Simple Econometric Model

Nelson & Siegel (1985)<sup>33</sup> proposed as a model for the forward intensity:

$$(3.1) \quad f(v) = \beta_1 + \beta_2 e^{-\alpha v} + \beta_3 \alpha v e^{-\alpha v}$$

The implied yield curve follows as an averaged integral using (2.3):

$$(3.2) \quad y(v) = \beta_1 + \beta_2 \left( \frac{1 - e^{-\alpha v}}{\alpha v} \right) + \beta_3 \left( \frac{1 - e^{-\alpha v}}{\alpha v} - e^{-\alpha v} \right)$$

and the implied present value function follows using (2.1):

$$(3.3) \quad p(v) = \exp \left( -\beta_1 v - (\beta_2 + \beta_3) \left( \frac{1 - e^{-\alpha v}}{\alpha} \right) + \beta_3 v e^{-\alpha v} \right)$$

Diebold & Li (2006)<sup>34</sup> extend this Nelson-Siegel model by incorporating a change process through calendar time  $t$ . This enables them to forecast future yield curves.

Compared with Nelson-Siegel, Smith & Wilson (2001)<sup>35</sup> start the other way around. They propose a model for the present value function, from which the yield and forward intensity function follow. The specification of this present value function needs a special type of function, known as Wilson function, that we will focus on next.

### 4. Wilson Function

The Wilson function  $W(u, v)$  can be specified as:

$$(4.1) \quad W(u, v) = e^{-\alpha(u+v)} H(u, v) = e^{-\alpha u} H(u, v) e^{-\alpha v}$$

where  $H(u, v)$  is the heart of the Wilson function:

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<sup>33</sup>Nelson, Charles R & Siegel, Andrew F, 1987. "Parsimonious Modelling of yield curves". *The Journal of Business*, University of Chicago Press vol. 60 (4) 473-489, October.

<sup>34</sup>Francis X. Diebold & Canlin Li (2006). "Forecasting the term structure of government bond yields". *Journal of Econometrics* vol. 130 337-364.

<sup>35</sup>A. Smith & T. Wilson (2001). "Fitting yield curves with long term constraints". London: Bacon & Woodrow



$$\begin{aligned}
H(u, v) &= \alpha \min(u, v) - \exp(-\alpha \max(u, v)) \cdot \sinh(\alpha \min(u, v)) \\
(4.2) \quad &= \alpha \min(u, v) + \frac{e^{-\alpha(u+v)} - e^{-\alpha|u-v|}}{2} \\
&= \frac{\alpha(u+v) + e^{-\alpha(u+v)} - \alpha|u-v| - e^{-\alpha|u-v|}}{2}
\end{aligned}$$

Here  $\alpha$  and  $\omega$  are parameters that have a dimension reciprocal to that of the time duration to maturity  $u$  and  $v$  that we take the year, and measured as number of days divided by 365.25.

The parameter  $\omega$  denotes the ultimate forward intensity and takes the value  $\log(1.042)$  in case the ultimate forward rate equals 4.2%. The parameter  $\alpha$  controls the speed of convergence to this asymptotic level.

This  $H$ -function and its first two derivatives happen to be continuous at  $v=u$ :

$$(4.3) \quad H(u, v) = \alpha \min(u, v) - \exp(-\alpha \max(u, v)) \cdot \sinh(\alpha \min(u, v))$$

Differentiation with respect to  $v$  gives:

$$(4.4) \quad \frac{dH(u, v)}{dv} = G(u, v) = \begin{cases} \alpha - \alpha e^{-\alpha u} \cosh(\alpha v) & v \leq u \\ \alpha e^{-\alpha v} \sinh(\alpha u) & u \leq v \end{cases}$$

For the second order derivative we have:

$$(4.5) \quad \frac{d^2H(u, v)}{dv^2} = \alpha^2 H(u, v) - \alpha^3 \min(u, v)$$

However, the third derivative shows a discontinuity at  $u=v$ .

## 5. Matrices and vectors

Matrices and vectors will be boldface. Transposition is indicated by a prime and  $\circ$  denotes element-wise multiplication of conformable matrices.  $\mathbf{1}$  and  $\mathbf{0}$  will denote column vectors with all components equal to 1 and 0 respectively, and of appropriate order.

We introduce a vector  $\mathbf{u}$  for the  $m$  observed durations to maturity as well as an  $m \times n$  cash-flow matrix  $\mathbf{C}$  that may contain zeros:

$$(5.1) \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} \quad c_{ij} \geq 0$$

Nonlinear functions of vectors will indicate by square brackets the component-wise operation as in:

$$(5.2) \quad \mathbf{d} = \exp[-\boldsymbol{\omega}\mathbf{u}] = \begin{bmatrix} e^{-\omega u_1} \\ e^{-\omega u_2} \\ \vdots \\ e^{-\omega u_m} \end{bmatrix} \quad p[\mathbf{u}] = \begin{bmatrix} p(u_1) \\ p(u_2) \\ \vdots \\ p(u_m) \end{bmatrix} \quad \sinh[\boldsymbol{\omega}\mathbf{u}] = \frac{1}{2} \begin{bmatrix} e^{\omega u_1} - e^{-\omega u_1} \\ e^{\omega u_2} - e^{-\omega u_2} \\ \vdots \\ e^{\omega u_m} - e^{-\omega u_m} \end{bmatrix}$$

We need an auxiliary matrix  $\mathbf{Q} = \mathbf{d}_\Delta \mathbf{C}$  where the subscript  $\Delta$  denotes transforming a column vector into a diagonal matrix such that  $\mathbf{d}_\Delta \mathbf{1} = \mathbf{d}$ . Furthermore we have three column vectors with  $n$  components:

$$(5.3) \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \mathbf{Q}'\mathbf{1} = \mathbf{C}'\mathbf{d}$$

Here  $\mathbf{p}$  contains the  $n$  observed market prices for the  $n$  financial instruments that will be contrasted with the  $m$  components of the present values in  $p[\mathbf{u}]$ .

The data can be stored in an  $(m+1) \times (n+1)$  tableau containing  $\mathbf{C}$  bordered by  $\mathbf{u}$  and the transpose of  $\mathbf{p}$ :

$$(5.4) \quad \begin{bmatrix} \mathbf{p}' \\ \mathbf{C} \quad \mathbf{u} \end{bmatrix}$$

Without loss of generality we may order the rows of this tableau according to the components of  $\mathbf{u}$  such that there holds  $u_1 < u_2 < \dots < u_m$ . Likewise the columns of this tableau can be ordered such that  $\mathbf{C}$  will be as upper-triangular as possible. Such a canonical format will be useful for validation purposes but is not of any importance for the mathematical formulations.

Zero-rows in  $\mathbf{C}$  can be deleted from the tableau without loss of generality. In case of non-deletion this will imply zero components in the output vector  $\mathbf{Q}\mathbf{b}$  at the appropriate places.

The tableau, whether canonical or not, can be normalized by dividing the columns by the appropriate component of  $\mathbf{p}$ , that is post-multiplying with the inverse of  $\mathbf{p}_\Delta$ :

$$(5.5) \quad \begin{bmatrix} \mathbf{1}' \\ \mathbf{C}\mathbf{p}_\Delta^{-1} \quad \mathbf{u} \end{bmatrix}$$

In applied work the normalized cash-flow matrix  $\mathbf{C}\mathbf{p}_\Delta^{-1}$  will often be denoted by  $\mathbf{C}$  right-away. In case of zero-coupon bonds, the canonical format makes  $\mathbf{C}$  a diagonal matrix that can be normalized to the identity matrix  $\mathbf{I}$  resulting in a canonical normalized tableau:

$$(5.6) \quad \begin{bmatrix} \mathbf{p}' \\ \mathbf{I} \quad \mathbf{u} \end{bmatrix}$$

Of course, this case does not need a data tableau, but just  $\mathbf{u}$  and  $\mathbf{p}$ . In what follows we will not assume a canonical or normalized format for the data, such that the exposition holds in full generality.

## 6. Wilson Matrix and H-matrix

Now we are in the position to display

$$(6.1) \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} \quad \mathbf{W}(\mathbf{v}, \mathbf{u}) = \begin{bmatrix} W(v_1, u_1) & W(v_1, u_2) & \cdots & W(v_1, u_m) \\ W(v_2, u_1) & W(v_2, u_2) & \cdots & W(v_2, u_m) \\ \vdots & \vdots & & \vdots \\ W(v_k, u_1) & W(v_k, u_2) & \cdots & W(v_k, u_m) \end{bmatrix} = \mathbf{W}'(\mathbf{u}, \mathbf{v})$$

$$(6.2) \quad \mathbf{W}(v, \mathbf{u}) = [W(v, u_1) \quad W(v, u_2) \quad \cdots \quad W(v, u_m)] = \mathbf{W}'(\mathbf{u}, v)$$

$$(6.3) \quad \mathbf{W}(\mathbf{u}, \mathbf{u}) = \mathbf{W} = \mathbf{d}_\Delta \mathbf{H} \mathbf{d}_\Delta = \mathbf{H} \circ \mathbf{d} \mathbf{d}'$$

The symmetric matrices  $\mathbf{W}$  and  $\mathbf{H}$  will be positive definite as soon as  $\mathbf{u}$  contains distinct positive components. Implementation with  $\mathbf{H}$  is simpler as it only depends on  $\alpha$  and not on  $\omega$ .

## 7. Smith-Wilson Present Value Function

This function, also known as discount pricing function, can be displayed as:

$$(7.1) \quad p(v) = e^{-\omega v} + \mathbf{W}(v, \mathbf{u}) \mathbf{C} \mathbf{b} = e^{-\omega v} + e^{-\omega v} \mathbf{H}(v, \mathbf{u}) \mathbf{Q} \mathbf{b}$$

We can form a set of equations by having  $v$  the values of  $\mathbf{u}$ :

$$(7.2) \quad p[\mathbf{u}] = \exp[-\omega \mathbf{u}] + \mathbf{W} \mathbf{C} \mathbf{b} = \mathbf{d} + \mathbf{W} \mathbf{C} \mathbf{b} = \mathbf{d} + \mathbf{d}_\Delta \mathbf{H} \mathbf{d}_\Delta \mathbf{C} \mathbf{b} = \mathbf{d} + \mathbf{d}_\Delta \mathbf{H} \mathbf{Q} \mathbf{b}$$

Pre-multiplication with the transpose of  $\mathbf{C}$  gives  $n$  linear equations in  $\mathbf{b}$ :

$$(7.3) \quad \mathbf{C}' p[\mathbf{u}] = \mathbf{C}' \mathbf{d} + \mathbf{C}' \mathbf{W} \mathbf{C} \mathbf{b} = \mathbf{q} + \mathbf{Q}' \mathbf{H} \mathbf{Q} \mathbf{b}$$

We have  $\mathbf{p}$  as the market observable counterpart of  $\mathbf{C}' p[\mathbf{u}]$

$$(7.4) \quad \mathbf{p} = \mathbf{q} + \mathbf{Q}' \mathbf{H} \mathbf{Q} \mathbf{b}$$

From this follows the solution for  $\mathbf{b}$ :

$$(7.5) \quad \mathbf{b} = (\mathbf{Q}' \mathbf{H} \mathbf{Q})^{-1} (\mathbf{p} - \mathbf{q})$$

This solution depends on  $\omega$  through  $\mathbf{Q}$  and  $\mathbf{q}$  as well as on  $\alpha$  through  $\mathbf{H}$ .

The value for  $\alpha$  will be determined through convergence requirements.

## 8. Smith-Wilson for Zero-Coupon Bonds

When  $m=n$ , the cash-flow matrix  $\mathbf{C}$  may be taken as the identity matrix and we are in the zero-coupon bond case. The present value function simplifies as:

$$(8.1) \quad p(v) = e^{-\omega v} (1 + \mathbf{H}(v, \mathbf{u}) \tilde{\mathbf{b}}) \quad \text{where} \quad \tilde{\mathbf{b}} = \mathbf{d} \circ \mathbf{b}$$

and the calculation for the coefficient vector

$$(8.2) \quad \tilde{\mathbf{b}} = \mathbf{H}^{-1}(\mathbf{p} \circ \exp[\omega \mathbf{u}] - \mathbf{1})$$

## 9. Smith-Wilson Yield and Forward Intensity Function

From (7.1) the yield intensity function follows as:

$$(9.1) \quad y(v) = \frac{-\log p(v)}{v} = \omega - \frac{\log(1 + \mathbf{H}(v, \mathbf{u}) \mathbf{Qb})}{v}$$

The forward intensity function follows as:

$$(9.2) \quad f(v) = \frac{-d \log p(v)}{dv} = \omega - \frac{d \log(1 + \mathbf{H}(v, \mathbf{u}) \mathbf{Qb})}{dv} = \omega - \frac{\mathbf{G}(v, \mathbf{u}) \mathbf{Qb}}{1 + \mathbf{H}(v, \mathbf{u}) \mathbf{Qb}}$$

where the components of the row vector  $\mathbf{G}(v, \mathbf{u})$  follow from (4.4).

As  $H(u, v)$  has a continuous second order derivative, we conclude that the Smith-Wilson present value and yield curve are sufficiently smooth at the nodes given by the observed liquid maturities. However, the forward intensity curve is less smooth as it does not have a continuous second order derivative at these nodes.

## 10. Zero Spot Intensity

When  $v \leq \min(\mathbf{u})$  we have from (4.4):

$$(10.1) \quad \mathbf{G}'(v, \mathbf{u}) = \mathbf{G}(\mathbf{u}, v) = \alpha \mathbf{1} - \alpha \cosh(\alpha v) \exp[-\alpha \mathbf{u}]$$

For  $v \downarrow 0$  we get:

$$(10.2) \quad \mathbf{H}'(0, \mathbf{u}) = \mathbf{H}(\mathbf{u}, 0) = \mathbf{0} \quad \mathbf{G}(\mathbf{u}, 0) = \alpha \mathbf{1} - \alpha \exp[-\alpha \mathbf{u}]$$

From this the zero spot intensity follows from (9.2) as:

$$(10.3) \quad y(0) = f(0) = \omega - \alpha \mathbf{1}' \mathbf{Qb} + \alpha \exp[-\alpha \mathbf{u}] \mathbf{Qb}$$

## 11. Analysis of Convergence to Ultimate Forward Intensity

When  $v \geq U = \max(\mathbf{u})$  we have from (4.3) and (4.4):

$$(11.1) \quad \mathbf{H}(\mathbf{u}, v) = \alpha \mathbf{u} - e^{-\alpha v} \sinh[\alpha \mathbf{u}] \quad \mathbf{G}(\mathbf{u}, v) = \alpha e^{-\alpha v} \sinh[\alpha \mathbf{u}]$$

Now, the upper end of the forward intensity function reduces to:

$$(11.2) \quad f(v) = \omega + \frac{\alpha}{1 - \kappa \cdot e^{\alpha v}} \quad v \geq U$$

where  $\kappa$  is a quasi-constant that depends on  $\alpha$  (and  $\omega$ ) but not on  $v$ :

$$(11.3) \quad \kappa = \frac{1 + \alpha \mathbf{u}' \mathbf{Q} \mathbf{b}}{\sinh[\alpha \mathbf{u}' \mathbf{Q} \mathbf{b}]}$$

When  $\alpha$  is such that  $\kappa=0$  we have  $f(v) = \omega + \alpha$  irrespective of the value of  $v$  and the ultimate forward intensity  $f(\infty)$  will not approach  $\omega$ .

The value of  $\alpha$  is controlled by requirements on convergence speed and will automatically avoid an  $\alpha$  that makes  $\kappa=0$ .

Adopting a convergence period  $S = \max(40, 60 - U)$  implies a point of convergence  $T$

$$(11.4) \quad T = U + S = \max(U + 40, 60)$$

We view the gap at the point of convergence  $T$  as a function of  $\alpha$ :

$$(11.5) \quad g(\alpha) = |f(T) - \omega| = \frac{\alpha}{|1 - \kappa e^{\alpha T}|}$$

and formulate the problem of determining  $\alpha$  as a nonlinear minimization problem:

Minimize  $\alpha$

with respect to  $\alpha$

subject to the two inequality conditions:

$$(1) \quad \alpha \geq a \text{ where the lower bound } a = 0.05$$

$$(2) \quad g(\alpha) \leq \tau$$

A heuristic solution strategy could be to test:

if  $\alpha = a$  implies  $g(\alpha) \leq \tau$  then optimal  $\alpha = a$

else search for  $\alpha > a$  such that  $g(\alpha) = \tau$

Please take care not to rewrite the second inequality  $g(\alpha) \leq \tau$  as  $\alpha \leq \tau |1 - \kappa e^{\alpha T}|$ .

Without the first inequality this might favour a false root for  $\alpha$  approaching the value 0.

## 14.B. Fitting the term structure to bond prices and swap rates

With the Smith-Wilson technique the term structure can be fitted to all the different financial instruments that may be eligible as basis for assessing the risk-free interest rate curve.

Each set of instruments that is taken as input, is defined by:

- vector of the market prices of  $n$  instruments at valuation date,
- vector of the  $m$  different cash payment dates up to the last maturity, and
- $m \times n$  matrix of the cash-flows on the instruments at these dates.

We will now look at this input when the term structure is fitted to zero coupon bond rates, coupon bond rates and par swap rates.

Instruments	Market prices $\mathbf{p}$	Cash payment dates $\mathbf{u}$	Cash-flow matrix $\mathbf{C}$
zero coupon bonds	<ul style="list-style-type: none"> <li>• Market prices of the <math>n</math> input instruments, given as the percent amount of the notional amount</li> <li>• The market prices of the zero coupon input bonds translate at once into spot rates for input maturities</li> </ul>	<ul style="list-style-type: none"> <li>• The cash payment dates are the maturity dates of the <math>n</math> zero coupon input bonds (i.e. <math>m=n</math>)</li> </ul>	<ul style="list-style-type: none"> <li>• An <math>n \times n</math> matrix with entries: <ul style="list-style-type: none"> <li>- <math>c_{ij} = 1</math> for <math>i=j</math>,</li> <li>- <math>c_{ij} = 0</math> else.</li> </ul> </li> <li>• <math>\mathbf{C}</math> reduces to the identity matrix.</li> </ul>
coupon bonds	<ul style="list-style-type: none"> <li>• Market prices of the <math>n</math> coupon input bonds, given as the percent amount of the notional amount of the bond.</li> </ul>	<ul style="list-style-type: none"> <li>• The cash payment dates are, in addition to the maturity dates of the input bonds all coupon dates.</li> </ul>	<ul style="list-style-type: none"> <li>• An <math>m \times n</math> matrix with entries: <ul style="list-style-type: none"> <li>- <math>c_{ij} = r_c(i)/s</math>, <math>j &gt; t(i)</math></li> <li>- <math>c_{i,t(i)} = 1 + r_c(i)/s</math>,</li> <li>- <math>c_{ij} = 0</math>, <math>j &lt; t(i)</math>,</li> </ul>           where <math>r_c(i)</math> is the coupon rate of <math>i^{\text{th}}</math> bond, and <math>s</math> is the settlement frequency         </li> </ul>
par swap rates	<ul style="list-style-type: none"> <li>• The market prices of the <math>n</math> par swap input</li> </ul>	<ul style="list-style-type: none"> <li>• The cash payment dates are, in addition to the</li> </ul>	<ul style="list-style-type: none"> <li>• An <math>m \times n</math> matrix with entries:</li> </ul>

	<p>instruments are taken as unit (i.e. 1).</p> <ul style="list-style-type: none"> <li>To receive the swap rate, a floating rate has to be earned, that can be swapped against the fixed rate. To earn the variable rate a notional amount has to be invested. At maturity, the notional amount is de-invested.</li> </ul>	<p>maturity dates of the swap agreements all swap rate payment dates.</p>	<ul style="list-style-type: none"> <li>- <math>c_{ij} = r_c(i)/s, j &gt; t(i)</math></li> <li>- <math>c_{i,t(i)} = 1 + r_c(i)/s,</math></li> <li>- <math>c_{ij} = 0, j &lt; t(i),</math></li> </ul> <p>where <math>r_c(i)</math> is the swap rate of agreement <math>i</math>, and <math>s</math> is the settlement frequency.</p>
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## 14.C. Numerical Illustration

With the data in the canonical normalized format as given on the next page and where the ultimate forward intensity  $\omega = \log(1.042)$  and convergence period  $S=40$ , the following results are obtained for the key parameters of the Smith-Wilson method:

<i>UFR</i>	4,2%	ultimate forward rate		<b>u</b>	<b>Qb</b>
$\omega$	0,0411	ultimate forward intensity		1	-2,045
				2	-0,528
$\kappa$	0,7379			3	3,375
				4	-4,119
<i>U</i>	20	last liquid point		5	3,831
<i>S</i>	40	convergence period		6	-1,583
<i>T</i>	60	convergence point		7	-1,036
				8	5,910
$ f(T) - \omega $	0,0001	gap at convergence		9	-11,183
$\tau$	0,0001	convergence tolerance		10	19,266
				11	-28,567
target	0,123760	minimize with respect to $\alpha$ .		12	27,689
$\alpha$	0,123760			13	-12,632
<i>a</i>	0,05	lowerbound $\alpha$		14	-2,724
				15	2,212
				16	10,182
				17	-17,203
				18	10,943
				19	-4,674
				20	2,314

With  $\alpha$ ,  $\omega$ , **u** and **Qb** the Smith-Wilson present value function can be evaluated for any maturity  $v$ :

$$p(v) = e^{-\omega v} (1 + H(v, \mathbf{u}) \mathbf{Qb})$$

The yield intensity follows as:

$$y(v) = \frac{-\log p(v)}{v}$$

and the annualized yield rate can be calculated as a fractional power of the present value function or as the exponential of the yield intensity:

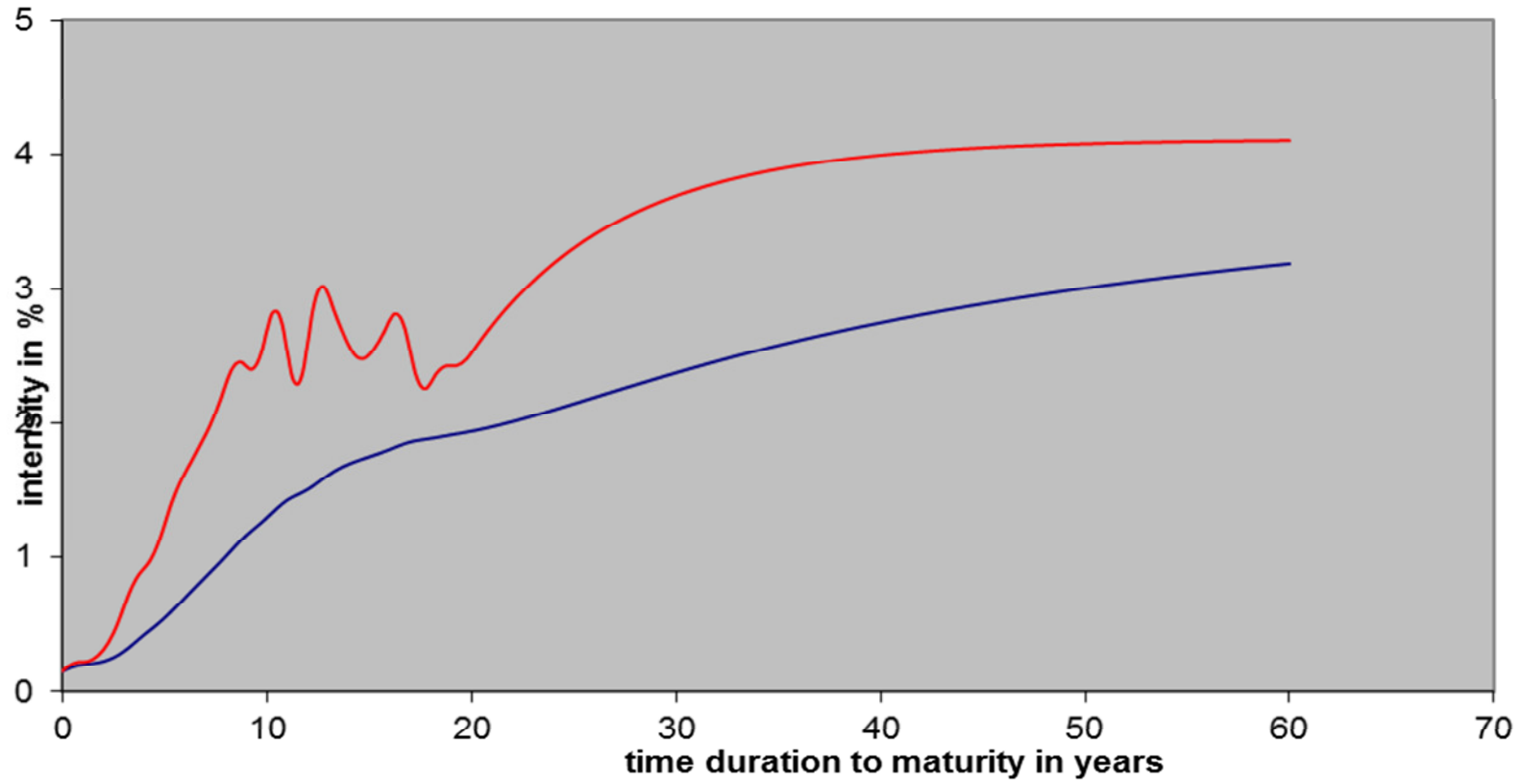
$$(p(v))^{-1/v} - 1 = \exp(y(v)) - 1$$

Besides the data tableau in canonical normalized format on the next pages, also a graph of the yield and forward intensity curve is displayed and a tabulation of yield intensity together with annualized yield rate for maturities from 0 up to 120 years.



transpose of market observed p																						
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
cash-flow matrix C																					<b>u</b>	
1,002	0,00225	0,003	0,00425	0,0055	0,007	0,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	1		
0	1,00225	0,003	0,00425	0,0055	0,007	0,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	2		
0	0	1,003	0,00425	0,0055	0,007	0,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	3		
0	0	0	1,00425	0,0055	0,007	0,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	4		
0	0	0	0	1,0055	0,007	0,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	5		
0	0	0	0	0	1,007	0,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	6		
0	0	0	0	0	0	1,0085	0,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	7		
0	0	0	0	0	0	0	1,01	0,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	8		
0	0	0	0	0	0	0	0	1,0115	0,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	9		
0	0	0	0	0	0	0	0	0	1,01275	0,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	10		
0	0	0	0	0	0	0	0	0	0	1,014	0,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	11		
0	0	0	0	0	0	0	0	0	0	0	1,01475	0,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	12		
0	0	0	0	0	0	0	0	0	0	0	0	1,01575	0,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	13		
0	0	0	0	0	0	0	0	0	0	0	0	0	1,0165	0,017	0,0175	0,018	0,01825	0,0185	0,01875	14		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,017	0,0175	0,018	0,01825	0,0185	0,01875	15		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,0175	0,018	0,01825	0,0185	0,01875	16		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,018	0,01825	0,0185	0,01875	17		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,01825	0,0185	0,01875	18		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,0185	0,01875	19		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,01875	20		

### yield & forward intensity curve



**Table of spot yield intensities (continuous curve)  
and annualized spot yield rates.**

maturity in years	yield intensity in %	yield rate in %	maturity in years	yield intensity in %	yield rate in %	maturity in years	yield intensity in %	yield rate in %
0	0,15043	0,15054						
1	0,19980	0,20000	41	2,77975	2,81875	81	3,42830	3,48774
2	0,22478	0,22503	42	2,80915	2,84898	82	3,43665	3,49639
3	0,29980	0,30025	43	2,83746	2,87810	83	3,44481	3,50483
4	0,42516	0,42607	44	2,86472	2,90614	84	3,45277	3,51307
5	0,55098	0,55250	45	2,89096	2,93315	85	3,46055	3,52112
6	0,70286	0,70533	46	2,91624	2,95918	86	3,46814	3,52898
7	0,85583	0,85951	47	2,94059	2,98425	87	3,47556	3,53667
8	1,01017	1,01529	48	2,96406	3,00842	88	3,48282	3,54418
9	1,16615	1,17298	49	2,98668	3,03173	89	3,48991	3,55152
10	1,29724	1,30569	50	3,00850	3,05421	90	3,49684	3,55870
11	1,43000	1,44027	51	3,02954	3,07590	91	3,50362	3,56572
12	1,50925	1,52070	52	3,04985	3,09683	92	3,51026	3,57260
13	1,61761	1,63077	53	3,06945	3,11704	93	3,51675	3,57932
14	1,69901	1,71353	54	3,08838	3,13657	94	3,52310	3,58590
15	1,75260	1,76805	55	3,10667	3,15543	95	3,52933	3,59235
16	1,80701	1,82343	56	3,12435	3,17367	96	3,53542	3,59866
17	1,86227	1,87972	57	3,14145	3,19131	97	3,54138	3,60484
18	1,88810	1,90604	58	3,15798	3,20838	98	3,54723	3,61089
19	1,91447	1,93291	59	3,17399	3,22490	99	3,55295	3,61682
20	1,94136	1,96032	60	3,18948	3,24089	100	3,55856	3,62264
21	1,97401	1,99362	61	3,20449	3,25638	101	3,56406	3,62834
22	2,01239	2,03278	62	3,21902	3,27139	102	3,56946	3,63393
23	2,05459	2,07584	63	3,23312	3,28595	103	3,57475	3,63941
24	2,09920	2,12139	64	3,24678	3,30006	104	3,57993	3,64478
25	2,14517	2,16834	65	3,26004	3,31376	105	3,58502	3,65006
26	2,19172	2,21591	66	3,27290	3,32705	106	3,59001	3,65523
27	2,23826	2,26350	67	3,28539	3,33996	107	3,59491	3,66031
28	2,28439	2,31068	68	3,29752	3,35249	108	3,59972	3,66529
29	2,32977	2,35712	69	3,30931	3,36468	109	3,60444	3,67019
30	2,37419	2,40260	70	3,32076	3,37652	110	3,60907	3,67499
31	2,41750	2,44696	71	3,33190	3,38803	111	3,61362	3,67971
32	2,45959	2,49009	72	3,34273	3,39923	112	3,61809	3,68434
33	2,50039	2,53192	73	3,35327	3,41013	113	3,62248	3,68889
34	2,53988	2,57240	74	3,36353	3,42073	114	3,62679	3,69337
35	2,57802	2,61154	75	3,37351	3,43106	115	3,63103	3,69776
36	2,61483	2,64932	76	3,38324	3,44112	116	3,63520	3,70208
37	2,65033	2,68576	77	3,39272	3,45093	117	3,63929	3,70633
38	2,68453	2,72089	78	3,40195	3,46048	118	3,64332	3,71050
39	2,71748	2,75474	79	3,41095	3,46979	119	3,64727	3,71460
40	2,74921	2,78735	80	3,41973	3,47888	120	3,65116	3,71864

## **15. Annex of Subsection 6.D. Rationale for the UFR calibration**

327. The most important economic factors explaining the long-term forward rate are long-term expected inflation and expected real interest rates. Two other components that can be seen to influence the long-term forward rate are the expected long-term nominal term premium and the long-term nominal convexity effect.
328. However, in order to have a robust and credible estimate for the UFR, the assessment shall be based only on the estimate of the expected inflation and the estimate of the expected short term real rate, the two components that are deemed to be most relevant, most stable and most reliable.
329. The term premium represents the additional return an investor may expect on risk-free long dated bonds relative to short dated bonds, as compensation for the longer term investment. This factor can have both a positive and a negative value, as it depends on liquidity considerations and on preferred investor habits.
330. The convexity effect arises due to the non-linear (convex) relationship between interest rates and the bond prices used to estimate the interest rates. This is a purely technical effect and always results in a negative component.
331. Both the term premium and the convexity premium can only be estimated from unobservable data in the extrapolated part of the curve. They would introduce a strong element of unpredictability in the estimation of the ultimate forward rate, and shall therefore be excluded from the UFR.

Table 1 shows inflation data for the OECD-countries in the period 1994 – 2013

Price indices (MEI): Consumer prices - Annual inflation

Table 1

Subject	Consumer prices - all items																				
Measure	Percentage change on the same period of the previous year																				
Frequency	Annual																				
Time	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	
Country																					
Australia	i	2	4,6	2,6	0,2	0,9	1,5	4,5	4,4	3	2,7	2,3	2,7	3,6	2,3	4,4	1,8	2,9	3,3	1,8	2,4
Austria	i	3	2,2	1,9	1,3	0,9	0,6	2,3	2,7	1,8	1,4	2,1	2,3	1,4	2,2	3,2	0,5	1,8	3,3	2,5	2
Belgium	i	2,4	1,5	2,1	1,6	0,9	1,1	2,5	2,5	1,6	1,6	2,1	2,8	1,8	1,8	4,5	-0,1	2,2	3,5	2,8	1,1
Canada	i	0,2	2,1	1,6	1,6	1	1,7	2,7	2,5	2,3	2,8	1,9	2,2	2	2,1	2,4	0,3	1,8	2,9	1,5	0,9
Chile	i	11,4	8,2	7,4	6,1	5,1	3,3	3,8	3,6	2,5	2,8	1,1	3,1	3,4	4,4	8,7	0,4	1,4	3,3	3	1,8
Czech Republic	i	10	9,1	8,8	8,5	10,7	2,1	3,9	4,7	1,8	0,1	2,8	1,9	2,6	3	6,3	1	1,5	1,9	3,3	1,4
Denmark	i	2	2,1	2,1	2,2	1,8	2,5	2,9	2,4	2,4	2,1	1,2	1,8	1,9	1,7	3,4	1,3	2,3	2,8	2,4	0,8
Estonia	i	..	..	..	..	8,7	3,3	4	5,7	3,6	1,3	3	4,1	4,4	6,6	10,4	-0,1	3	5	3,9	2,8
Finland	i	1,1	0,8	0,6	1,2	1,4	1,2	3	2,6	1,6	0,9	0,2	0,6	1,6	2,5	4,1	0	1,2	3,4	2,8	1,5
France	i	1,7	1,8	2	1,2	0,6	0,5	1,7	1,6	1,9	2,1	2,1	1,7	1,7	1,5	2,8	0,1	1,5	2,1	2	0,9
Germany	i	2,7	1,7	1,4	1,9	0,9	0,6	1,4	2	1,4	1	1,7	1,5	1,6	2,3	2,6	0,3	1,1	2,1	2	1,5
Greece	i	10,9	8,9	8,2	5,5	4,8	2,6	3,2	3,4	3,6	3,5	2,9	3,5	3,2	2,9	4,2	1,2	4,7	3,3	1,5	-0,9

Hungary	i	18,9	28,3	23,5	18,3	14,2	10	9,8	9,1	5,3	4,7	6,7	3,6	3,9	8	6	4,2	4,9	3,9	5,7	1,7
Iceland	i	1,6	1,7	2,3	1,8	1,7	3,2	5,1	6,4	5,2	2,1	3,2	4	6,7	5,1	12,7	12	5,4	4	5,2	3,9
Ireland	i	2,4	2,5	1,7	1,4	2,4	1,6	5,6	4,9	4,6	3,5	2,2	2,4	3,9	4,9	4,1	-4,5	-0,9	2,6	1,7	0,5
Israel	i	12,4	10	11,3	9	5,4	5,2	1,1	1,1	5,7	0,7	-0,4	1,3	2,1	0,5	4,6	3,3	2,7	3,5	1,7	1,6
Italy	i	4,1	5,2	4	2	2	1,7	2,5	2,8	2,5	2,7	2,2	2	2,1	1,8	3,3	0,8	1,5	2,8	3	1,2
Japan	i	0,7	-0,1	0,1	1,8	0,7	-0,3	-0,7	-0,8	-0,9	-0,2	0	-0,3	0,2	0,1	1,4	-1,3	-0,7	-0,3	0	0,4
Korea	i	6,3	4,5	4,9	4,4	7,5	0,8	2,3	4,1	2,8	3,5	3,6	2,8	2,2	2,5	4,7	2,8	2,9	4	2,2	1,3
Luxembourg	i	2,2	1,9	1,2	1,4	1	1	3,2	2,7	2,1	2	2,2	2,5	2,7	2,3	3,4	0,4	2,3	3,4	2,7	1,7
Mexico	i	7	35	34,4	20,6	15,9	16,6	9,5	6,4	5	4,5	4,7	4	3,6	4	5,1	5,3	4,2	3,4	4,1	3,8
Netherlands	i	2,8	1,9	2	2,2	2	2,2	2,3	4,2	3,3	2,1	1,2	1,7	1,2	1,6	2,5	1,2	1,3	2,3	2,5	2,5
New Zealand	i	1,7	3,8	2,3	1,2	1,3	-0,1	2,6	2,6	2,7	1,8	2,3	3	3,4	2,4	4	2,1	2,3	4	1,1	1,1
Norway	i	1,4	2,4	1,2	2,6	2,3	2,3	3,1	3	1,3	2,5	0,5	1,5	2,3	0,7	3,8	2,2	2,4	1,3	0,7	2,1
Poland	i	33	28	19,8	14,9	11,6	7,2	9,9	5,4	1,9	0,7	3,4	2,2	1,3	2,4	4,2	3,8	2,6	4,2	3,6	1
Portugal	i	5,4	4,2	3,1	2,3	2,6	2,3	2,9	4,4	3,6	3,2	2,4	2,3	3,1	2,5	2,6	-0,8	1,4	3,7	2,8	0,3
Slovak Republic	i	13,4	9,8	5,8	6,1	6,7	10,6	12	7,3	3,1	8,6	7,5	2,7	4,5	2,8	4,6	1,6	1	3,9	3,6	1,4
Slovenia	i	21	13,5	9,9	8,4	7,9	6,2	8,9	8,4	7,5	5,6	3,6	2,5	2,5	3,6	5,7	0,9	1,8	1,8	2,6	1,8
Spain	i	4,7	4,7	3,6	2	1,8	2,3	3,4	3,6	3,1	3	3	3,4	3,5	2,8	4,1	-0,3	1,8	3,2	2,4	1,4
Sweden	i	2,2	2,5	0,5	0,7	-0,3	0,5	0,9	2,4	2,2	1,9	0,4	0,5	1,4	2,2	3,4	-0,5	1,2	3	0,9	0
Switzerland	i	0,9	1,8	0,8	0,5	0	0,8	1,6	1	0,6	0,6	0,8	1,2	1,1	0,7	2,4	-0,5	0,7	0,2	-0,7	-0,2

Turkey	i	105,2	89,1	80,4	85,7	84,6	64,9	54,9	54,4	45	21,6	8,6	8,2	9,6	8,8	10,4	6,3	8,6	6,5	8,9	7,5	
United Kingdom	i	2	2,6	2,5	1,8	1,6	1,3	0,8	1,2	1,3	1,4	1,3	2,1	2,3	2,3	3,6	2,2	3,3	4,5	2,8	2,6	
United States	i	2,6	2,8	2,9	2,3	1,6	2,2	3,4	2,8	1,6	2,3	2,7	3,4	3,2	2,9	3,8	-0,4	1,6	3,2	2,1	1,5	
G7	i	2,2	2,3	2,3	2	1,3	1,5	2,3	2,1	1,3	1,8	2	2,4	2,4	2,2	3,3	-0,1	1,4	2,6	1,9	1,3	
OECD – Europe	i	8,4	8,5	7,3	7	6,9	5,3	5,5	5,3	4,7	3	2,4	2,4	2,5	2,6	3,8	1,2	2,3	3,2	2,9	1,9	
OECD – Total	i	4,8	6	5,6	4,8	4,2	3,6	4	3,6	2,8	2,4	2,4	2,6	2,6	2,5	3,7	0,5	1,9	2,9	2,2	1,6	
Non-OECD Member Economies	Brazil	i	2075	66	15,8	6,9	3,2	4,9	7	6,8	8,5	14,7	6,6	6,9	4,2	3,6	5,7	4,9	5	6,6	5,4	6,2
	China	i	24,1	17,1	8,3	2,8	-0,8	-1,4	0,4	0,7	-0,8	1,2	3,9	1,8	1,5	4,8	5,9	-0,7	3,3	5,4	2,6	2,6
	India	i	10,2	10,2	9	7,2	13,2	4,7	4	3,8	4,3	3,8	3,8	4,2	5,8	6,4	8,3	10,9	12	8,9	9,3	10,9
	Indonesia	i	8,5	9,4	8	6,2	58,5	20,5	3,7	11,5	11,9	6,8	6,1	10,5	13,1	6,4	10,2	4,4	5,1	5,4	4,3	6,7
	Russian Federation	i	307,	197,	47,9	14,7	27,8	85,7	20,8	21,5	15,8	13,7	10,9	12,7	9,7	9	14,1	11,7	6,9	8,4	5,1	6,8
	South Africa	i	8,9	8,7	7,4	8,6	6,9	5,2	5,3	5,7	9,5	5,7	-0,7	2,1	3,2	6,2	10	7,2	4,1	5	5,7	5,8

data extracted on 22 Jul 2014 08:00 UTC (GMT) from OECD Stat

332. Singapore, Malaysia, Thailand, Hong Kong and Taiwan are not included in the list from the OECD database. The data for these currencies can be found in Table 2 and are taken from Eco-Win (Reuters) database.

Table 2: Inflation 1994-2010 Certain Asian Countries

Country		Year								
Consumer Prices		1994	1995	1996	1997	1998	1999	2000	2001	
Hong Kong,	CPI, Total, Index, 2004-05=100	9,6 %	7,0 %	6,7 %	5,2 %	-1,6 %	-4,0 %	-2,1 %	-3,6 %	
Malaysia,	Total, Index, 2005=100	3,5 %	3,2 %	3,3 %	2,9 %	5,3 %	2,5 %	1,2 %	1,2 %	
Singapore,	All items, Index, 2009=100	2,9 %	0,8 %	2,0 %	2,0 %	-1,4 %	0,7 %	2,1 %	-0,6 %	
Thailand,	Total, Index, 2007=100	4,7 %	7,5 %	4,7 %	7,7 %	4,3 %	0,6 %	1,5 %	0,7 %	
Taiwan,	Total, Index, 2006=100	2,7 %	4,6 %	2,5 %	0,3 %	2,1 %	0,1 %	1,6 %	-1,7 %	
Country		Year								
Consumer Prices		2002	2003	2004	2005	2006	2007	2008	2009	2010
Hong Kong,	CPI, Total, Index, 2004-05=100	-1,5 %	-1,9 %	0,3 %	1,4 %	2,3 %	3,8 %	2,0 %	1,3 %	3,3 %
Malaysia,	Total, Index, 2005=100	1,7 %	1,2 %	2,1 %	3,2 %	3,1 %	2,4 %	4,4 %	1,1 %	2,0 %
Singapore,	All items, Index, 2009=100	0,4 %	0,7 %	1,3 %	1,3 %	0,8 %	3,7 %	5,5 %	-0,5 %	4,6 %
Thailand,	Total, Index, 2007=100	1,7 %	1,7 %	3,0 %	5,8 %	3,5 %	3,2 %	0,4 %	3,5 %	3,0 %
Taiwan,	Total, Index, 2006=100	0,8 %	-0,1 %	1,6 %	2,2 %	0,7 %	3,3 %	1,3 %	-0,2 %	1,2 %

333. During the last 15 years, Turkey has been categorised by OECD as a high inflation country<sup>36</sup>. Turkey's inflation target is also higher (5-7.5% for the period 2009 - 2012) than in other countries. Mexico, Brazil, and India have had persistent high inflation rates in the last 15

<sup>36</sup> <http://stats.oecd.org/index.aspx>



years. South Africa has had high inflation rates during the decade from 1994 to 2003, a drop to negative inflation in 2004 and rising inflation rates up to 2008. Moreover, Mexico's inflation target for 2010 is 3%, Brazil's national monetary council has set the inflation target at 4.5% plus or minus two percentage points for this year and 2011, South African's central bank has set the upper end of its inflation target at 3%-6% and in India the central bank does not follow a policy of targeting inflation.

334. Based on this data basis, notwithstanding the fact that the inflation in Mexico and South Africa has gone down, for the time being we would rank these five currencies in the high inflation group.
335. Hungary, Greece, Indonesia, Singapore and Iceland are also possible candidates for the high inflation group. However, Greece is part of the Euro-zone and for the other countries the deviations to the average inflation rate are far more moderate (and at least for Singapore also only a quite recent phenomenon) than those for the other high inflation countries. Therefore, Hungary, Singapore and Iceland continue classified in the standard inflation category.
336. Japan, having deflation in the period since 1994, is an obvious candidate for the "low inflation"-group. Switzerland can also be evaluated likewise. This is due to the fact that historically relatively low inflation rates can be observed and that Switzerland is particularly attractive in the international financial markets (exchange rate conditions, liquidity, "safe haven"<sup>37</sup>...). For these reasons, lower inflation assumptions are applied for the Swiss currency.
337. The estimate covers one-year inflation rate 70 - 100 years from now. It is arbitrary to say whether the inflation differences we see today and have seen the last 20 years will persist 100 years into the future. However, historical evidence and current long-term interest rates indicated that it is reasonable to have three groups of currencies with different inflation assumptions. The standard inflation rate is set to 2% per anno. To allow for deviations up and down to the standard inflation rate, an adjustment to the estimate of  $\pm 1$  percentage point was applied for the high inflation group and the low inflation group respectively. This adjustment of 1 percentage point was applied to the estimated inflation rate for these specific countries based on differences in current long-term interest rates (30Y), observed historical differences between the average interest rate and differences in short term inflation expectations.

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<sup>37</sup>Peter Kugler and Beatrice Weder, "Why Are Returns on Swiss Franc Assets So Low? Rare events may solve the puzzle", *Applied Economics Quarterly* 51/3 (2005), pages 231-246

338. The following grouping is used for the estimated expected long-term inflation rate:
- a. Standard inflation rate set to 2%:  
Euro-zone, UK, Norway, Sweden, Denmark, USA, Poland, Hungary, Iceland, Czech Republic, Bulgaria, Latvia, Lithuania, Estonia, Romania, Canada, Australia, Korea, China, Singapore, Malaysia, Thailand, Hong Kong and Taiwan
  - b. High inflation rate set to 3%:  
Turkey, Brazil, Mexico, South Africa and India
  - c. Low inflation rate set to 1%:  
Japan, Switzerland
339. One can expect that the real rates should not differ substantially across economies as far out as 100 years from now. Elroy Dimson, Paul Marsh and Mike Staunton provide a global comparison of annualized bond returns over the last 111 years (1900 to 2010) for the following 19 economies: Belgium, Italy, Germany, Finland, France, Spain, Ireland, Norway, Japan, Switzerland, Denmark, Netherlands, New Zealand, UK, Canada, US, South Africa, Sweden and Australia<sup>38</sup>.
340. In an earlier publication, the same authors compared the real bond returns from the second versus the first half of the 20<sup>th</sup> century for the following 12 economies: Italy, Germany, France, Japan, Switzerland, Denmark, Netherlands, UK, Canada, US, Sweden and Australia<sup>39</sup>. The average real bond return over the second half of the 20<sup>th</sup> century was computed as annually 2.3% (compared to -1.1% for the first half of the 20<sup>th</sup> century).
341. In light of the above data, 2.2% is an adequate estimate for the expected real interest rate.

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<sup>38</sup>Elroy Dimson, Paul Marsh and Mike Staunton. Credit Suisse Global Investment Returns Yearbook 2011".

<sup>39</sup>Dimson, E., Marsh, P. and Staunton, M. (2000), Risk and Return in the 20th and 21st Centuries". Business Strategy Review, 11: 1–18. See Figure 4 on page 5.

## **16. Annex 8.D. Calculation of the relative composition (in percentages) of the reference portfolios**

The most updated information for the calculation of the reference portfolios is the one contained at market level in EIOPA Stress Test 2014 (template BS+ of the baseline). This information refers to 31-12-2013.

Further than the method and assumptions detailed in section 8.B, the following operational criteria have been applied

Data have been aggregated at market level keeping in mind the currency of the asset. In case of individual assets with a negative value, they have been excluded.

Where information on the currency of the asset is not available, it has been assumed that the relative composition (in percentages) of the market aggregates for all assets is an appropriate proxy for the reference portfolio of assets expressed in the relevant currency.

Assets declared as belonging to credit quality steps 5-6, have been fully allocated to credit quality 5.

Unrated exposures have been allocated proportionally to the rated assets.

Durations at market level have been calculated as the weighted average of each individual duration, using as weights the market value of the asset.

Exposures expressed in percentages have been rounded. In case the total exposure after rounding is not 100%, the rounding differences (+/-1%) have been allocated to the largest exposure.

In absence of the necessary information, for the preparatory phase the reference portfolio of market yield indices for assets other than central government and central bank bonds, is the same for the calculation of the currency volatility adjustment and for the calculation of the country specific increase.

The appropriate differentiation among both portfolios will be implemented in light of the reporting templates during the preparatory phase.

## 17. Annex to subsection 9.C.2. and 9.C.3. Calculation of the cost of downgrade (CoD) and probability of default (PD)

### Legal Context

The two components Cost of Downgrade (CoD) and Probability of Default (PD) are required by Article 77c (2-a i and ii) (Calculation of the matching adjustment) of Directive 2009/138/EC as amended by the Omnibus II directive 2014/51/EU (OMD-II), in conjunction with Article 51 (Risk-corrected spread, for volatility adjustment) and 54 (4) (Calculation of the matching adjustment) of the Delegated Regulation. Furthermore, OMD-II recital (31) and Delegated Regulation recitals (22) and (23) apply.

The Cost of Downgrade (CoD) is defined as the present value of costs resulting from future downgrade, expressed as spreads in base points over the risk-free interest rates. According to Article 54 (4) (a) the cash flow pattern does not change, according to (b) the replacing asset belongs to the same asset class as the replaced asset, and according to (c) the replacing asset has the same credit quality step or a better one as the replaced asset.

As described below, the same approach applies to the Probability of Default (PD) with the appropriate modifications.

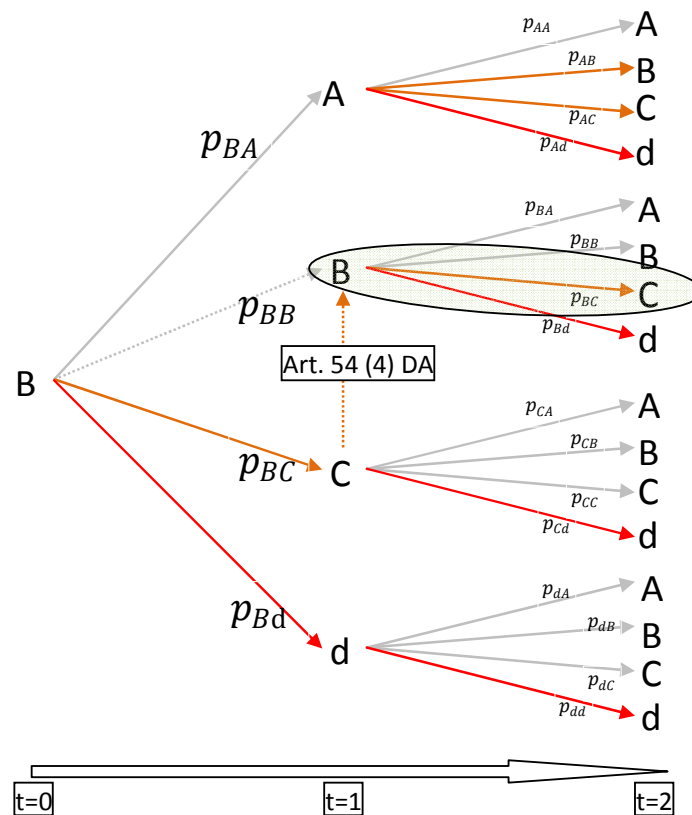
### The Three components of a present value

$$PV = \sum_{t=1}^T \frac{\text{CashFlow}_t \cdot \text{Probability}(\text{Cashflow})}{(1 + \text{InterestRate}_t)^t}$$

### Probability

Looking from  $t = 0$  ("today"), the probability for a downgrade event from  $X$  to  $Y$  to occur between time  $t = t_0$  and  $t = t_1$  is given as the probability for the bond to be in CQS  $X$  at time  $t = t_0$  and then to end in CQS  $Y$  at time  $t = t_1$ .

**Example: Downgrade from B to C between  $t = 1$  and  $t = 2$  for a B Bond at inception  $t = 0$**



The probability for being in CQS B at time  $t = 1$  is determined by all the paths leading to B in  $t = 1$ . For the above example, where we only consider the initial CQS B, the path without replacement would be  $B \rightarrow B \rightarrow C$ . However, due to the requirement of Article 54 (4) of the Delegated Regulation to *replace* bonds that have been downgraded by a bond of the CQS it was in before the downgrade event, we could have also come to B at time  $t = 1$  via the path  $B \rightarrow C \xrightarrow{\text{Art.54 (4)}} B$ . So, the total probability to have a downgrade event between  $t = 1$  and  $t = 2$  is given by  $(P_{BB} + P_{BC}) \cdot P_{BC}$ .

Hence, the replacement requirement of Article 54 (4) of the Delegated Regulation leads to the following 'change' in that transition matrix which determines the starting credit quality step for the year in which the cost of the downgrading event is accounted:

$$T = \begin{pmatrix} P_{AA} & P_{AB} & P_{AC} & P_{Ad} \\ P_{BA} & P_{BB} & P_{BC} & P_{Bd} \\ P_{CA} & P_{CB} & P_{CC} & P_{Cd} \\ P_{dA} & P_{dB} & P_{dC} & P_{dd} \end{pmatrix}$$

$$\xrightarrow{\text{Art. 54 (4) Delegated Regulation}} \begin{pmatrix} P_{AA} + P_{AB} + P_{AC} & \xleftarrow{\text{Art. 54 (4) DR}} & \xleftarrow{\text{Art. 54 (4) DR}} & P_{Ad} \\ & P_{BA} & P_{BB} + P_{BC} & \xleftarrow{\text{Art. 54 (4) DR}} P_{Bd} \\ & P_{CA} & P_{CB} & P_{CC} \\ & P_{dA} & P_{dB} & P_{dC} \\ & & & P_{dd} \end{pmatrix} = Q$$

The original transition matrix  $T$  is retained for those probabilities regarding the transitions in the year the cost accounting is done.

This means, the probability for a downgrade from  $B$  at  $t = 1$  to  $C$  at  $t = 2$  is given by the probability of being in credit quality step  $B$  at  $t = 1$  (regarding possible upgrading events due to Art. 54 (4) of the Delegated Regulation between  $t = 0$  and  $t = 1$ ), multiplied by the probability  $P_{BC}$  of transitioning from credit quality step  $B$  at  $t = 1$  to  $C$  at  $t = 2$ . In matrix notation, this can be expressed by the matrix multiplication of  $Q$  for the possible paths from  $t = 0$  to  $t = 1$  with  $T$  for the possible paths from  $t = 1$  to  $t = 2$ .

More general, for a downgrade event to be accounted for in year  $m$  (i.e. between  $t = m$  and  $t = m + 1$ ), we consider the matrix  $Q$  the first  $m$  times and then once the matrix  $T$ . Thus, the probabilities to be used for a downgrade event in year  $m$  (i.e. between  $t = m$  and  $t = m + 1$ ) are all contained in the matrix  $Q^m T$ .

### Zero bond cash flow $(-1), 0, \dots, 0, (1 + r_t)^t$

By Article 54 (4) of the Delegated Regulation, the cash flow in case of downgrade is defined as the difference in market values of the original (higher) credit quality and the new (lower) credit quality. There is no specific requirement for the case of upgrade, the case of staying in the same credit quality or for the case of defaulting. The defaulting case is considered in the separate component for PD (probability of default).

The corresponding market values change over time. The cash flows are derived from zero bonds with investment  $(-1)$  at inception  $t = 0$  and final payment  $(1 + r_t)^t$  at maturity. The compound interest rate  $r_t$  is based on the financial instrument considered to be risk-free once adjusted. For Solvency II purposes, this is considered to be the basic risk-free interest rate structure.

### Discount factor

The discount factor  $1/(1 + \text{InterestRate}_t)^t$  considers the risk-free spot rate.

The above considerations give rise to the following nutshell description.

## Cost of Downgrade and Probability of Default in a nutshell

### Input Data

Transition Matrix  $T = (p_{X,Y})_{X,Y \in CQS}$  for the  $n$ -element set  $CQS$  of credit quality steps including default state denoted by "d" (note that  $p_{dX} = 0$  and  $p_{dd} = 1$  because  $d$  is considered an absorbing state) and relevant portions  $R_c$  for credit quality steps  $c \in CQS$ . Any explicit reference to economic sectors or other granularity buckets is dropped, because Article 54 (4) of the Delegated Regulation does not require costs of transitions between economic sectors or other granularity buckets to be considered. However, the following calculation needs to be done within each of those buckets not explicitly mentioned here.

For the first publications in 2015, the portions  $R_c$  are simply provided by EIOPA. During the review phase in 2015, EIOPA plans to investigate in a calibration of the  $R_c$  based on market data.

### Cost of Downgrade

Based on the basic risk-free interest rate term structure  $(r_M)_{M=1\dots30}$ , the market value of a zero bond of maturity  $M$  at time  $m$  is given by

$$MV_M(m) = \frac{(1 + r_M)^M}{(1 + f_{m,M})^{M-m}},$$

where the forward rates  $f_{m,M}$  are derived on an arbitrage-free basis:

$(1 + r_m)^m (1 + f_{m,M})^{M-m} = (1 + r_M)^M$ . This provides the following closed formula for the market value of the risk-free reference instrument:

$$MV_M(m) = (1 + r_m)^m.$$

The market value of the risky instruments in CQS  $c$  is defined based on a fixed portion  $R_c$  as a portion of the risk-free instrument and given by

$$MV_{c,M}(m) = R_c^{\frac{M-m}{15}} \cdot (1 + r_m)^m.$$

The portion is a certain percentage  $R_c^M$  of the market value of the risk-free reference instrument at inception and increases to 100% at maturity. The factors are applied having in mind 15 years maturity.

A downgrade at time  $m$  from credit quality step  $X$  to  $Y > X$  results in the following cost:

$$CoD_{(X,Y),M}(m) := MV_{X,M}(m) - MV_{Y,M}(m) > 0.$$

Define the following strictly upper triangular matrix (an upgrade or stay is not accounted for):

$$C_M^{(m)} := \left( \left\{ \begin{array}{l} CoD_{(X,Y),M}(m) \cdot p_{X,Y} \text{ for } Y \neq \text{default} \\ (1 - \text{RecoveryRate}) \cdot MV_{X,M}(m) \cdot p_{X,Y} \text{ for } Y = \text{default} \end{array} \right\}_{(X<Y) \in CQS} \right)$$

Define the matrix  $Q$  according to the replacement requirement of Article 54 (4) of the Delegated Regulation

$$(q_{XY})_{X,Y \in CQS} := \left. \left\{ \begin{array}{l} p_{XY} \quad \text{for } X > Y \text{ and } Y = n \text{ (lower triangle and rightmost column)} \\ \sum_{k=i}^{n-1} p_{i,k} \quad \text{for } X = Y \leq n \text{ (Art. 54 (4) DR) (main diagonal)} \\ 0 \quad \text{for } X < Y < n \text{ (upper triangle except rightmost column)} \end{array} \right\} \right.$$

The following matrix contains the expected cash flows representing the expected cost of downgrade for bonds in the credit quality step in  $CQS$  of original maturity  $M$  at times  $m = 1, \dots, M$ .

$$\begin{pmatrix} CoD_{\text{best quality},M}(1) & \cdots & CoD_{\text{best quality},M}(M) \\ \vdots & \ddots & \vdots \\ CoD_{\text{lowest quality},M}(1) & \cdots & CoD_{\text{lowest quality},M}(M) \\ CoD_{\text{default},M}(1) = 0 & \cdots & CoD_{\text{default},M}(1) = 0 \end{pmatrix} := \bigcup_{m=1}^M \underbrace{\left( Q^{m-1} C_M^{(m)} \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{pmatrix} \right)}_{\text{column vector}},$$

where  $\bigcup_{m=1}^M (\cdot)$  shall denote the concatenation (to the right) of column vectors into a matrix. In base points,  $CoD_{c,M}^{(bp)}$  is solved from the following equation. Note  $CoD_{c,M}^{(bp)} = 0$  if  $CoD_{c,M}(m) = 0$  for all  $c, m$ .

$$\frac{1}{(1 + r_M + CoD_{c,M}^{(bp)})^M} = \frac{1}{(1 + r_M)^M} \left( 1 - \sum_{m=1}^M \frac{CoD_{c,M}(m)}{(1 + r_m)^{m-0.5}} \right)$$

## Probability of Default in a nutshell

The computation of the probability of default in *base points as spread over the basic risk-free rate* is done completely consistently with the above approach. There is no Article 54 (4) requirement to replace downgraded bonds along the way. Hence, the only difference is to use the original transition

matrix  $T$  instead of the “twisted Article 54 (4) matrix”  $Q$  and to use the column vector  $\begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$  instead

of  $\begin{pmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{pmatrix}$ . Rename  $CoD$  to  $PD$  in this case. The other special case corresponds to the RecoveryRate

term, which is given by Article 54 (2) of the Delegated Regulation as 30% of the market value of the bond.

For the risk-correction of cash flows to be considered in the matching adjustment, the *probability of default is the total probability for a zero bond's final payment at maturity not to occur*. This probability is independent of market values and just given by the rightmost column of the matrix powers  $T^m$ .



## Reducing computational and numerical complexity

Please note that  $C_M^{(m)}$  is strictly upper triangular. This might help to further reduce complexity if needed. One can setup an internal table of all the values  $(1 + r_m)^m$  and  $R_c^m$  for  $m = 1, \dots, M$ . The market values are then just given by the product of two entries of this fixed-value table.

Furthermore, the matrix powers  $Q^m$  and  $T^m$  can be saved in an internal (three-dimensional) array.

The matrix  $Q$  excluding the last row and column is lower triangular with non-zero values on the main diagonal (unless "stay or upgrade" would both be impossible for any CQS). That is, the diagonal consists of the Eigenvalues  $\lambda_c$  of the matrix  $Q$  which is immediate from the characteristic polynomial decomposing into linear terms of the form  $(\lambda - \lambda_c)$ . Write  $Q = S^{-1} * \text{diag}(\lambda_c)_{c \in CQS} * S$ , then  $Q^m = S^{-1} \text{diag}(\lambda_c^m)_{c \in CQS} S$ , where the columns of  $S$  are the corresponding left-Eigenvectors.

## A remark about probability in continuous time and why it has not been used here

In this notation, one could – in theory – also define matrix powers for non-integral times  $t$  by

$$Q^t := S^{-1} \text{diag}(\lambda_c^t) S.$$

However, the use of the continuous version of powers of  $Q$  should carefully consider whether continuous downgrade events with immediate upgrade make sense in the specific application context. Even if one would consider integrals instead of sums, downgrades would still be discrete jumps between a finite number of rating classes or credit quality steps. This could be different if spreads were considered to continuously change without regard to a rather limited number of rating categories or credit quality steps. The choice taken in this approach stays away from this complexity in order to create consistency with the mechanics behind the creation of transition matrices.

## A remark about intra-year chains of rating changes

In real life, if a financial instrument receives a downgrade with negative forecast, it is not unlikely that the same instrument receives a second downgrade within the same year. The approach taken here would not "see" this chain of rating changes, because it only looks at discrete points  $t = 0, t = 1, \dots, t = M$ .

However, this would only be influential on the result if there is an upgrade event followed by a downgrade event in that chain of rating changes, because this downgrade event would have to be accounted for. But it is not accounted for, because it would not be recognized if one only opens the "black box" at the next point in time. Since these events are quite unlikely to occur, we disregard the difference stemming from this simplified view.

If the chain consists of only downgrading events, there is almost no difference at all, because the CoD cashflows are defined as differences between market values:

$$CoD_{X \rightarrow Y} + CoD_{Y \rightarrow Z} = (MV_X - MV_Y) + (MV_Y - MV_Z) = MV_X - MV_Z = CoD_{X \rightarrow Z}.$$

The only difference would stem from the different points in time and therefore the different interest/forward rates concerned. But again, this simplification has been considered to be of negligible materiality. However, in theory, this can be recognized within this model.

### Transition matrix implementing the rebalancing requirement after a downgrade event

Define the lower triangular matrix  $Q$  according to the replacement requirement of Art. 54 (4) of Delegated Regulation

$$(q_{XY})_{X,Y \in CQS} := \left\{ \begin{array}{ll} p_{XY} & \text{for } X > Y \text{ and } Y = n \text{ (lower triangle and rightmost column)} \\ \sum_{k=i}^{n-1} p_{i,k} & \text{for } X = Y \leq n \text{ (Art. 54 (4) DA) (main diagonal)} \\ 0 & \text{for } X < Y < n \text{ (upper triangle except rightmost column)} \end{array} \right\}$$

$$= \begin{pmatrix} \sum_{k=1}^{n-1} p_{1,k} & \leftarrow 0 & \cdots & \leftarrow 0 & p_{1d} \\ p_{21} & \sum_{k=2}^{n-1} p_{2,k} & \leftarrow 0 & \vdots & p_{2d} \\ p_{31} & p_{32} & \sum_{k=3}^{n-1} p_{3,k} & \leftarrow 0 & p_{3d} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ p_{n-1,1} & \cdots & p_{n-2,n-1} & p_{n-1,n-1} & p_{n-1,d} \\ p_{d1} = 0 & p_{d2} = 0 & \cdots & p_{d,n-1} = 0 & p_{dd} = 1 \end{pmatrix}.$$

## 18. Annex to subsection 9.C.4 Background on the Danish covered bond treatment

1. *Nykredits Realkreditindeks* includes a representative extract of the Danish covered bond market. The index includes both covered bonds with short and long maturities.
2. A single index which covers all maturities is preferred over a more granular approach e.g. mapping exposures to two indices with maturity 3 years and 30 years. Such a mapping will include major expert judgement on the split of insurance undertakings holdings of short and long duration covered bonds.
3. The use of a single index reflects better the exposures of the Danish insurance sector as a whole than an attempt to map exposures in to two buckets. It should also be noted that the *Nykredits Realkreditindeks* is the index used as input for the covered bond component in the current Danish interest rate curve.
4. Historical data for the yield of *Nykredits Realkreditindeks* is given in the figure below. This data corresponds to the input  $R_{covered}^{DKK}$
5. The average yield to maturity of this covered bond index for the time period 1 September 2003 to 31 December 2014 is 3.86 %

